Information Bias in the Proxy Advisory Market*

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Abstract

We study an information sale problem in which a monopolist proxy advisor sells recommendations to a firm's shareholders for corporate voting. We find that even an unconflicted proxy advisor skews its recommendations based on its clients' beliefs or preferences. A novel bias-quantity relationship affects firm value. Under some parameters, shareholders' biased beliefs or preferences can lead shareholders to make more information purchases, which enhances their collective decision-making. Thus, firm value may increase despite the negative effects of biased proxy voting recommendations.

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When occasions present themselves, in which the interests of the people are at variance with their inclinations, it is the duty of the persons whom they have appointed to be the guardians of those interests, to withstand the temporary delusion, in order to give them time and opportunity for more cool and sedate reflection.

— Alexander Hamilton, *The Federalist Paper: No. 71*

Proxy advisors provide investors with analyses of and recommendations for voting on matters that are presented for a shareholder vote. They have wielded increasing influence over institutional investors over the past three decades. Especially since 2003, when the SEC required institutional investors to vote on all matters on the corporate proxy and to disclose their votes to the beneficial owners of their holdings (Securities and Exchange Commission, 2003), institutional investors have substantially increased their use of proxy advisors due to the size and diversity of their investment portfolios. For example, as of 2019, the largest proxy advisor, namely, the Institutional Shareholder Services (ISS), serves over 2,000 institutional clients and covers more than 20,000 firms.

Because of the nature of their business as information intermediaries, proxy advisors are expected to provide information that supports informed corporate voting. However, as with other information intermediaries, we are also concerned about the quality of their information, because their interests are not necessarily aligned with those of their clients. In particular, we ask what the proxy advisors' contribution to informed corporate voting is and where their limitations are. Do proxy advisors provide institutional clients with accurate, unbiased information about the corporate proposals? Without an economic stake in the corporation, will their proxy voting services increase corporate values? How will the potential conflicts of interest that are inherent in some proxy advisors' business models affect the quality of their information and corporate values? When the interests of their clients are at variance with their inclinations, will the proxy advisors cater to these clients or adhere to the principle of acting as a clear-minded third party?

In this paper, we provide a model for addressing these relevant questions. The central finding of the paper is that a proxy advisor maximizes its profits only when it maximizes the perceived value of its recommendations by its clients; hence, even an unconflicted proxy advisor will skew its recommendations based on the biases of its client base (such as asset managers). A novel bias-quantity relationship affects firm value. In some cases, the buy-side biases can lead to more information purchases by shareholders, which enhances their collective voting decision-making. Therefore, the consequent firm value may even be higher than that in a bias-free economy, despite the negative effects of the biased proxy voting recommendations.

Our model is based on the strategic voting literature (e.g., Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996). Many aspects of our model, and the equilibrium that we focus on, are adapted from Malenko and Malenko (2019). In our model, a proxy advisor sells its binary voting recommendations to shareholders, who, in turn, use this information to vote on a proposal at a firm meeting. The proposal merits implementation only when the world is in a good state; however, the state is uncertain to all agents ex ante. The proxy advisor can commit to a policy to issue its voting recommendations, and once it receives its private signal regarding the state, it makes the recommendations based on the prespecified rule. After observing the proxy advisor's information price and the recommendation issue policy, each shareholder decides whether to purchase the recommendation. Then all shareholders vote simultaneously based on their information sets, and the proposal is implemented if it is approved by the majority of shareholders.

This paper focuses on the information bias in proxy advisory markets and its implications for firm value. We define a voting recommendation as *unbiased* if and only if the advice is consistent with the more likely state given the evidence, namely, the proxy advisor recommends voting in favor of a proposal only if its private signal suggests that the state is more likely to be good.

If the implementation of the proposal were solely determined by the recommendation, then an unbiased recommendation would maximize firm value. This represents the most efficient use of the proxy advisor's information and further supports the use of unbiasedness as an important criterion for evaluating information quality in the proxy advisory market.

Starting from the unbiased voting recommendation, we consider three potential sources of bias: (1) the conflicted proxy advisor, (2) shareholders' interests being misaligned with firm value maximization, and (3) shareholders holding incorrect beliefs about the state. We call the first a supply-driven force and the remaining two demand-driven forces.

As a meaningful benchmark, first, we present an economy that is free from all potential sources of bias, namely, the proxy advisor does not have conflicts of interest, shareholders have correct beliefs about the state, and they aim to maximize firm value. In this ideal setting, the proxy advisor offers unbiased voting recommendations to shareholders. This suggests that we can reasonably expect high-quality voting recommendations from the proxy advisor even though it is only a for-profit information intermediary.

Meanwhile, although the recommendation is unbiased, because of the free-rider problem among shareholders, the monopolistic proxy advisor rations the supply of its recommendation; that is, not all shareholders are informed of the recommendation in equilibrium. This facilitates the preservation of the value of the proxy advisor's information and maximizes its profits. Therefore, the resultant firm value is lower than that in the above ideal case, in which the advisor's information is efficiently used to maximize firm value.

Then we investigate the potential sources of information bias in the proxy advisory market. Our finding is intuitive: proxy advisors with conflicts of interest (e.g., providing consulting services to the firm) make biased recommendations. Relative to the benchmark, the value of the recommendation is lower, and it sells at a higher volume; that is, more shareholders are informed of the recommendation in equilibrium. While more shareholders cast their votes based on the recommendation, the bias in it ultimately hurts the collective decision-making, thereby leading to a lower firm value compared to that in the benchmark. Such negative consequences of the conflicted proxy advisor are consistent with the empirical findings in Li (2016).

Next, we study the potential bias that arises from the demand side. We find that the bias of the proxy advisor's clients can be passed on to its voting recommendations and that, in some cases, the advisor's slanted advice can be associated with more information acquisition by shareholders, thereby enhancing the collective decision-making and increasing firm value beyond that in the benchmark. Ironically, this is true even though in the benchmark, each and every shareholder holds correct prior beliefs and goes great lengths to maximize firm value.

Specifically, we consider the demand-driven bias that arises from shareholders' preferences or beliefs. For example, shareholders may not aim to maximize firm value, or they may hold incorrect prior beliefs about the state. The former occurs when fund managers are more sensitive to losses than gains due to career concerns (Haigh and List, 2005; Bodnaruk and Simonov, 2016) or when mutual funds support the proposal that is initiated by managers due to business ties with the firm (Cvijanović et al., 2016). The latter occurs when, for example, institutional investors are affected by public opinion and become overoptimistic about the proposal (Aggarwal et al., 2015).

Our model delivers the following insights. First, the proxy advisor skews its recommendations to cater to its clients' preferences or beliefs. For example, if shareholders are loss averse or overly pessimistic about the state, the proxy advisor is biased toward making negative recommendations, that is, suggesting to vote against the proposal. Why doesn't an unconflicted proxy advisor issue unbiased recommendations? This is because the proxy advisor maximizes its profits if and only if it maximizes the expected utility that is *perceived* by its clients. When contemplating their information-purchase decisions, shareholders expect the proxy advisor to recommend that they vote for the proposal only when the advisor's private signal strongly indicates a good state. To cater to such a prior voting tendency, the proxy advisor raises the bar on sending positive recommendations and hence issues negative recommendations more often. Overall, shareholders evaluate voting recommendations based on their own preferences and subjective prior beliefs. An evaluation determines shareholdersâĂŹ willingness to pay for recommendations and, thus, the proxy advisorâĂŹs profits. According to Hayne and Vance (2019, p. 975), "the proxy advisor representatives we interviewed emphasized that their guidelines come from an aggregation of institutional investor preferences."

Surprisingly, firm value can increase upon the biased voting recommendations relative to firm value in the benchmark. Firm value is determined by a novel bias-quantity relationship. The proxy advisor caters to its clients' preferences or beliefs by skewing its recommendation.

However, in some cases, these biases can be associated with more shareholders purchasing the recommendations and casting informed votes. While the former bias effect impairs firm value, the latter quantity effect facilitates its increase. For some parameter regions, the latter quantity effect can prevail, thereby resulting in a higher firm value.

Third, this novel bias-quantity relationship arises from the interesting interplay between two frictions: the free-riding incentives among shareholders and the proxy advisor's monopolistic selling strategy. Let us consider the benchmark at which shareholders would vote against the proposal according to their prior (e.g., the true prior suggests that the state is more likely to be bad). If we were in a single-shareholder economy, the shareholder always would be pivotal, namely, the shareholder would be able to influence the implementation of the proposal; in this case, she must purchase the recommendation. However, in our strategic voting game, shareholders do not buy the recommendation with probability one. Otherwise, a shareholder can simply free ride on others' informed votes, and the proxy advisor's recommendation is valueless.

Now, with the introduction of the demand-driven bias, if the buy side becomes more averse to the loss or overly pessimistic, shareholders' conservative prior voting tendency can be strengthened; that is, they are more willing to vote against the proposal according to their prior. This suggests that shareholders' votes are less likely to be split, the likelihood of being pivotal declines, and the free-riding incentive increases, thereby depressing their willingness to buy the recommendation. Consequently, the proxy advisor must lower its information price. For the monopolistic proxy advisor, lowering its price corresponds to increasing the supply of its recommendation simultaneously to maintain its optimal profits. Therefore, more information acquisition can occur in equilibrium. If this "quantity effect" prevails, more informed votes may result in a higher firm value despite the proxy advisor issuing biased voting recommendations.

Finally, our paper also sheds new light on the debate related to proxy advisors. It suggests that a proxy advisor may only cater to the interests of its actual payers, that is, institutional investors, and does not care about the preferences of their actual beneficiaries. For example, if a fund manager is loss averse because of career concerns or reputation costs but the fund's investors are loss neutral, the proxy advisor sends a voting recommendation that is only desirable for the fund manager. If the interests of the fund manager and the fund's investors are highly divergent, the biased recommendations can hurt firm value. This also implies that a fund's subscription to the voting recommendations cannot be regarded as evidence that it votes in the best interest of its clients and that its fiduciary duties are fulfilled.[1]

Similarly, if shareholders have incorrect prior beliefs, the proxy advisor tends to issue recommendations that *confirm* these subjective beliefs, namely, if shareholders are misinformed, the proxy advisor has no incentive to correct their false beliefs, and the recommendation that is issued by the advisor only exacerbates this problem. In practice, this feature can be either a boon or a bane. For example, a misinformed or a slanted proxy advisor (if such misinformation or slant can be captured by its prior information) will not hurt the shareholder or firm value. However, if shareholders are misinformed, the proxy advisor has little incentive to guide them "to withstand the temporary delusion."

Our study is related to three streams of literature. First, our paper is part of the growing literature on information provision in financial markets. In their seminal papers, Admati and Pfleiderer (1986, 1988) analyze the sale of information to investors who in turn trade on the information in financial markets. These models share an important characteristic: an agent that becomes more informed imposes a negative externality upon others due to the competition among agents. Thus, in equilibrium, the information seller has an incentive to dilute the sold information. This insight has been supported in more general settings; Bergemann and Morris (2019) show that it is often optimal for the information designer to selectively obfuscate information.

We focus on the sale of information used for shareholder voting. In contrast to the competitive environment in a speculative market, a shareholder's informed voting has a positive externality on other shareholders. That is, each agent becoming more informed results in a collectively more informed voting decision. Hence, in the voting context, the information seller may sell its information of the highest quality. We contribute to this literature by

[1]For example, according to former SEC commissioner Daniel M. Gallagher, ISS requested and received a "no action" relief from the SEC staff. This response suggests that relying on the proxy advisory firm can ensure that an institutional investor satisfies its fiduciary duty to vote in the best interest of its clients (Gallagher, 2014).

showing that a for-profit information seller may provide the most accurate information in financial markets, namely, under some conditions, the distinct pursuits of the proxy advisor and shareholders can be aligned in the proxy advisory market.

The most relevant paper to ours is Malenko and Malenko (2019), who also study how the proxy advisor affects corporate voting. However, they focus on shareholders' trade-off between the information purchase from the proxy advisor and private information acquisition. Their key insight is that, because of the crowding-out effect, the presence of the proxy advisor increases firm value only if the precision of its recommendation is sufficiently high. Instead we focus on the information bias in the proxy advisory market, which, to the best of our knowledge, has not been studied in the literature. In addition, our proxy advisor is more sophisticated than that in Malenko and Malenko (2019) as our proxy advisor strategically determines not only the recommendation price but also the recommendation issuance policy, which is an additional and more realistic feature that is suitable for our research question. Levit and Tsoy (2019) also study the proxy advisor but focus on its one-size-fits-all recommendations.

A second stream of related research is on corporate voting. Theoretical studies include Maug (1999), Maug and Yilmaz (2002), Bond and Eraslan (2010), Brav and Mathews (2011), Levit and Malenko (2011), Eso et al. (2015), Bar-Isaac and Shapiro (2019), and Cvijanovic et al. (2017), among others. Our study differs from these theoretical studies in that we focus on the proxy advisor, which is a salient institutional feature of corporate voting. In addition, a growing empirical literature also examines the proxy advisors. For example, scholars consider the potential conflicts of interest that are inherent in the proxy advisors' business (Li, 2016), the influence that proxy advisors wield over their clients (Bethel and Gillan, 2002; Cai et al., 2009; Iliev and Lowry, 2014; Malenko and Shen, 2016), and the informational role of proxy advisors (Alexander et al., 2010; Ertimur et al., 2013; Aggarwal et al., 2015). Our framework facilitates understanding of the empirical findings and generates policy implications. We discuss several policy implications in Section 4.

Finally, our paper is related to the literature on media bias (for a survey, see Gentzkow et al. (2015)). Similar to the markets for news, we show that in the proxy advisory market, shareholders tend to favor the recommendations that have biases that match their own preferences or prior beliefs, and the proxy advisor finds it optimal to cater to its clients by sending biased recommendations. We contribute to this literature by documenting the novel bias-quantity relationship in affecting firm value in the context of corporate voting, which adds to our understanding of the substantive effects of information bias. Although the biased recommendations can reduce firm value because of their low information quality, more shareholders purchase the recommendations and cast informed votes, thereby enhancing collective decision-making and increasing firm value.

1 Model Setup

We adapt the setup in the standard common-value voting game (e.g., Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996) to study information bias in the proxy advisory market.

A shareholder meeting is held, at which a proposal is voted on, and one proxy advisor sells voting recommendations to shareholders . Denote the state $\theta \in \{0, 1\}$, where $\theta = 0$ represents a bad economy and $\theta = 1$ represents a good economy. The probability of the economy being good is μ_0, where $\mu_0 \in (0, 1)$, namely, $\Pr(\theta = 1) = \mu_0$.

There are four dates, that is, $t = \{0, 1, 2, 3\}$. At $t = 0$, the proxy advisor commits to a policy regarding the issuance of voting recommendations and sets the price to maximize its expected profits. At $t = 1$, each shareholder decides whether to subscribe to the proxy advisor's voting recommendations or to stay with her prior. At $t = 2$, the proxy advisor observes a private signal regarding the state. Shareholders who have purchased the proxy advisor's voting recommendation at $t = 1$ receive the voting advice. Then all shareholders vote simultaneously on the proposal. Finally, at $t = 3$, the proposal is implemented if and only if the majority of shareholders voted for it, and the payoffs are realized. We refer to $t = 0$ as the profit maximization stage, $t = 1$ as the information-purchase stage, and $t = 2$ as the voting stage. Figure 1 illustrates the sequence of events, and the model's variables are tabulated and explained in Table A1 in the appendix.

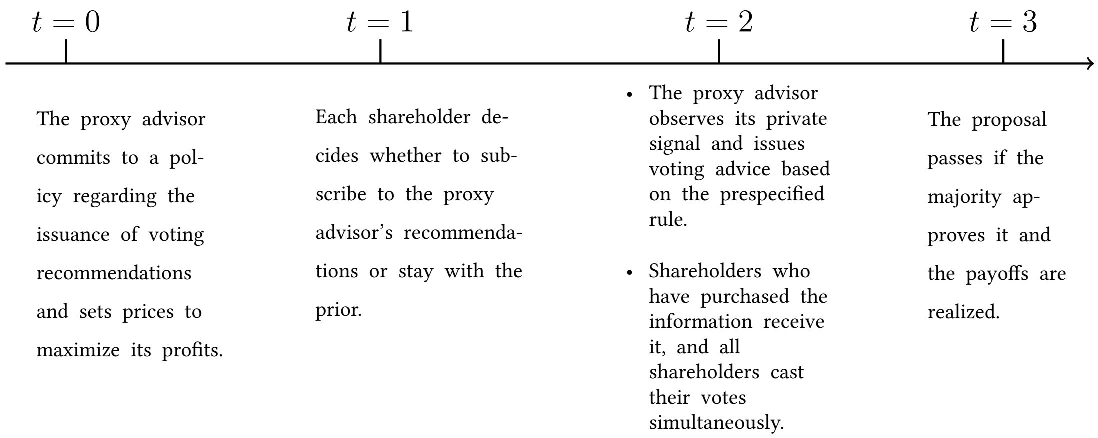


Figure 1: Time line of events

1.1 Shareholders and the proposal: information demand

Consider a firm that is owned by N shareholders, where N is odd and $N \geq 3$. We consider shareholders to be institutional investors, who often have significant holdings in the firm. Each shareholder owns one share, and the value of each share at the beginning of the economy is normalized to zero.

A shareholder meeting is held, at which a new proposal is voted on at $t = 2$. Shareholder i's voting decision is denoted by $v_i \in \{0, 1\}$, where $i \in \{1, ..., N\}$: $v_i = 1$ if shareholder i votes for the proposal, and $v_i = 0$ if she votes against it. All shareholders cast their votes simultaneously to maximize their expected utility (which will be specified shortly). The voting outcome is determined via the simple majority rule; that is, the proposal is accepted if and only if $\sum_{i=1}^{N} v_i \geq \frac{N+1}{2}$. $d \in \{0, 1\}$ denotes the voting outcome, with $d = 1$ representing acceptance of the proposal and $d = 0$ rejection.

The value of each share, which is denoted by $\psi(\theta, d)$, depends on the state and the voting outcome as follows:

$$\psi(\theta, d) = \begin{cases} 1 & \text{if } \theta = 1, d = 1, \\ -1 & \text{if } \theta = 0, d = 1, \\ 0 & \text{if } d = 0. \end{cases} \tag{1}$$

If the economy is good and the proposal is accepted ($\theta = 1$ and $d = 1$), the value of each share increases to 1. However, if the economy is bad but the proposal is accepted ($\theta = 0$ and $d = 1$), the firm incurs a loss, and its value per share decreases to -1. If the proposal is rejected, the share value does not change, regardless of the state.[2]

At $t = 1$, after observing the proxy advisor's recommendation issue policy and the price, each shareholder decides whether to subscribe to the proxy advisor's voting recommendations. q_i denotes shareholder i's information-purchase strategy, where $q_i \in [0, 1]$ for $i \in \{1, ..., N\}$. The variable q_i is the probability with which each shareholder purchases the proxy advisor's voting advice and can be regarded as the information demand. With shareholders being ex ante identical, we focus on the symmetric equilibrium (formally defined in Section 3). Therefore, the information-purchase strategy is the same across i in equilibrium; that is, $q_i = q$ for $i \in \{1, 2, ..., N\}$. If one shareholder purchases the voting recommendation, she will receive it at $t = 2$ and vote as an *informed* shareholder; otherwise, she only votes based on her prior and we refer to her as *uninformed*.

Motivated by Gentzkow et al. (2015), to study information bias in the proxy advisory market, we consider the potential drivers of bias from the demand side. In other words, the information bias that may arise from shareholders' prior beliefs or preferences. First, to capture the potential bias from shareholders' prior beliefs, we allow for the possibility that shareholders may not necessarily know the true prior of the good state μ_0; instead, they believe that the economy is in a good state with probability $\mu \in (0, 1)$. We allow $\mu \neq \mu_0$ in general. Shareholders are *overly optimistic* if $\mu > \mu_0$, and, conversely, they are *overly pessimistic* if $\mu < \mu_0$. Shareholders' subjective belief μ can be interpreted as the public opinion that widely affects the corporate voting by mutual funds (Aggarwal et al., 2015).

Second, shareholders' utility (gross of the potential fee that was paid for the voting rec-

[2]Many types of proposals, such as proposals about director elections, appointments of outside auditors, the issuance of new shares, the creation of equity-based compensation plans, amendments to the corporate charter or bylaws, major mergers and acquisitions, and ballot questions submitted in the form of advisory shareholder proposals, are relevant to share value (Yermack, 2010).

ommendations) is expressed as follows:

$$u(\theta, d) = \begin{cases} 1 & \text{if } \theta = 1, d = 1, \\ -\gamma & \text{if } \theta = 0, d = 1, \\ 0 & \text{if } d = 0, \end{cases} \tag{2}$$

where $\gamma > 0$. Compared with firm value per share in equation (1), shareholders' payoff is equal to the share value if and only if $\gamma = 1$. In other words, shareholders may not necessarily aim to maximize firm value. Such shareholder preferences are realistic in many settings. For example, mutual fund managers may be more sensitive to losses than gains because of career concerns (e.g., Haigh and List, 2005; Bodnaruk and Simonov, 2016), and this loss aversion preference is captured by $\gamma > 1$. Alternatively, if the proposal is initiated by managers, mutual funds tend to support it if they have business ties with the firm (e.g., Cvijanović et al., 2016). This is the case in which $\gamma < 1$. In addition, the incentives of socially responsible institutional investors are not necessarily aligned with firm-value maximization.

Shareholders aim to maximize their expected utility. Given their prior beliefs μ, shareholders hold the correct prior beliefs regarding the state and aim to maximize the expected firm value if and only if $\mu = \mu_0$ and $\gamma = 1$.

1.2 Proxy advisor and recommendations: Information supply

There exists a monopolistic proxy advisor (PA henceforth) that sells voting recommendations to shareholders. The PA's voting advice is based on its own research, which we model as a private signal $p \in \mathbb{R}$ regarding the state. $h(p|\theta)$ denotes the private signal's conditional probability density function:

$$h_1(p) \equiv h(p|\theta = 1) \text{ and } h_0(p) \equiv h(p|\theta = 0).$$

Let $H_1(\cdot)$ and $H_0(\cdot)$ be the cumulative distribution functions of $h_1(\cdot)$ and $h_0(\cdot)$, respectively. We assume that the conditional distributions satisfy the monotone likelihood ratio property (MLRP): the likelihood ratio $\frac{h_1(p)}{h_0(p)}$ is increasing with respect to p. In other words, larger values of the private signal p indicate that the state is more likely to be good. To rule out uninteresting corner solutions (as will be clear in the next section), we assume that $\frac{h_1(p)}{h_0(p)}$ spans from 0 to $+\infty$.

The PA receives this private signal at $t = 2$ and sends voting recommendations according to the prespecified rule. Although in practice, proxy advisors also send detailed reports on the voting item (the original private signal p), they must take a stand on the proposal when issuing voting advice, and this simple binary recommendation can be more influential. Therefore, following Malenko and Malenko (2019), we consider binary voting recommendations. Let $r \in \{0, 1\}$ denote the PA's voting advice. If $r = 1$, shareholders are advised to vote for the proposal, whereas if $r = 0$, they are advised to vote against it.

At $t = 0$, the PA must specify a voting recommendation scheme and sets the price to maximize profits. Suppose that the PA can commit to an information revelation rule that specifies how voting recommendations are revealed to buyers.[3] As the voting recommendation is binary, the PA uses a cutoff point scheme in sending the voting advice.[4] ϱ denotes the cutoff point: the PA advises shareholders to vote for the proposal if and only if the private signal p is sufficiently positive, that is, $r = 1$ if and only if $p \geq \varrho$. Therefore, conditional on the state, the respective probabilities of the PA sending a positive signal ($r = 1$) and a negative signal ($r = 0$) are as follows:

$$\pi_1 \equiv \Pr(r = 1|\theta = 1) = \int_{\varrho}^{+\infty} h_1(p)\,\mathrm{d}p = 1 - H_1(\varrho), \tag{3}$$

$$\pi_0 \equiv \Pr(r = 0|\theta = 0) = \int_{-\infty}^{\varrho} h_0(p)\,\mathrm{d}p = H_0(\varrho), \tag{4}$$

where $H_1(\cdot)$ and $H_0(\cdot)$ are the cumulative distribution functions of $h_1(\cdot)$ and $h_0(\cdot)$, respectively.

The PA also determines the price or fee for its voting recommendation at $t = 0$, denoted

[3]For example, the ISS publicly announced its voting principles for the 2020 proxy season. The principles underlie their approach for developing recommendations on management and shareholder proposals: https://www.issgovernance.com/file/policy/active/americas/US-Voting-Guidelines.pdf. This practice can be regarded as the proxy advisor's commitment to a recommendation scheme.

[4]If the PA sends a positive (negative) recommendation when observing $p = p'$, she always does so when $p \geq (\leq)\, p'$. Hence, the voting recommendation scheme must follow a threshold policy.

by $f \geq 0$. As each shareholder purchases the PA's voting recommendation with probability q, the expected number of shareholders who purchase the voting recommendation is qN. Therefore, the PA's profit from the proxy voting service is fqN.

In addition to the demand-side forces, we also consider the supply-side forces that may affect the information quality in the proxy advisory market. We study an economy in which the PA may have conflicts of interest. We model the conflicts of interest as an additional payoff or loss $\Phi \in \mathbb{R}$ to the PA if it issues a positive recommendation to shareholders ($r = 1$). For example, suppose that the proposal is management initiated and the PA also provides consulting services. Then the firm manager may use the PA's consulting services only if the PA recommends approval of the proposal; hence, $\Phi > 0$. Therefore, the PA's profits from the potential consulting service are $\Pr(r = 1)\Phi$. Only if $\Phi = 0$ is the PA unconflicted.

Considered together, the total profits of the PA from the proxy voting service and other conflicted services are $fqN + \Pr(r = 1)\Phi$. N is fixed, so we divide both sides of the PA's profit function by N and obtain its per capita form $f\,q + \Pr(r = 1)\,\varphi$, where $\varphi \equiv \frac{\Phi}{N}$. Thus, the PA's problem can be summarized as the selection of the price of the recommendation f and of the recommendation threshold ϱ that maximize the expected profits, as follows:

$$\max_{f,\varrho}\; f\,q + \Pr(r = 1)\,\varphi. \tag{5}$$

2 Equilibrium

We focus on a particular symmetric perfect Bayesian equilibrium:[5] all shareholders follow the same information-purchase strategy $q(f, \varrho) : \mathbb{R}^+ \times \mathbb{R} \to [0, 1]$, use the same voting strategy $v(\cdot) : \{\varnothing, 0, 1\} \to [0, 1]$ (where $\varnothing$ denotes that the shareholder does not purchase the PA's voting recommendations), and update their beliefs based on their priors and (if applicable) the PA's recommendations using the Bayes' rule wherever possible. The equilibrium is formally defined as follows.

Definition 1 (Equilibrium)**.** *An equilibrium is characterized by* $(f, \varrho, v(\cdot), q(\cdot, \cdot))$*, where* f *is*

[5] In the class of symmetric perfect Bayesian equilibria, this equilibrium is unique under some mild assumptions on equilibrium refinement (see Assumptions A1–A3 and Footnote 7).

the price of the PA's recommendation, ϱ is the recommendation threshold, the function $v(\cdot)$: $\{\varnothing, 0, 1\} \to [0, 1]$ is the probability of a shareholder voting for the proposal given her information set, and the function $q(\cdot, \cdot) : \mathbb{R}^+ \times \mathbb{R} \to [0, 1]$ is the probability with which a shareholder purchases the PA's recommendations. In an equilibrium, wherever possible, shareholders' posterior beliefs over states are determined by their priors and (if applicable) the PA's recommendations using Bayes' rule. Moreover,

1. *At $t = 2$ (the voting stage), the voting strategy $v(\cdot)$ maximizes each shareholder's expected utility (2) conditional on her information set and the voting strategies of all other shareholders;*
2. *At $t = 1$ (the information-purchase stage), the information-purchase strategy $q(f, \varrho)$ maximizes each shareholder's expected utility (net of the potential subscription fee for the recommendation); and*
3. *At $t = 0$ (the profit maximization stage), the price of the voting recommendation f and the threshold ϱ maximize the PA's expected profits.*

Next, we solve the model via backward induction.

2.1 Voting-stage equilibrium

At the voting stage, given any PA's selling strategy (f, ϱ) and shareholders' information-purchase strategy $q(f, \varrho)$, there exists a unique equilibrium.[6] The subgame equilibrium is fully characterized by Lemma A1 in the appendix. The following Lemma 1 only presents the subgame equilibrium on the equilibrium path, that is, the decision node that shareholders think is or will be in play.

Define

$$A_1 = A_1(\varrho) = \mu\pi_1 - \gamma(1-\mu)(1-\pi_0), \tag{6}$$

$$A_0 = A_0(\varrho) = -\mu(1-\pi_1) + \gamma(1-\mu)\pi_0. \tag{7}$$

[6]The uniqueness of the voting stage equilibrium is obtained by ruling out unrealistic equilibria in some off-equilibrium paths. See the proof of Lemma 1 in the appendix for a detailed discussion.

Here, A_1 denotes the gains of a pivotal shareholder if she follows the PA's recommendation to vote for the proposal rather than vote against it. Similarly, A_0 denotes the gains of a pivotal shareholder if she follows the PA's recommendation to vote against the proposal instead of voting for it.

Lemma 1 (Equilibrium voting strategy)**.** *In equilibrium, we have $0 < q < 1$, $A_1 > 0$, and $A_0 > 0$. At $t = 2$ (the voting stage), an informed shareholder votes according to the advice of the PA, and an uninformed shareholder votes for the proposal with a probability of $\tau \in (0,1)$, where τ is given as follows:*

$$\tau = \begin{cases} \frac{(1-M-2q)+\sqrt{(1-M)^2+4q^2M}}{2(1-q)(1-M)} & \text{if } \gamma \neq \frac{\mu}{1-\mu}, \\ \frac{1}{2} & \text{if } \gamma = \frac{\mu}{1-\mu}, \end{cases} \tag{8}$$

and M is defined as a function of ϱ as follows:

$$M = M(\varrho) = \sqrt[\frac{N-1}{2}]{\frac{A_0}{A_1}}. \tag{9}$$

In other words, $v(0) = 0$, $v(1) = 1$, and $v(\varnothing) = \tau$ at the voting stage in equilibrium.

Lemma 1 states that in equilibrium informed and uninformed shareholders coexist. Informed shareholders always vote according to the advice of the PA, whereas uninformed shareholders adopt a strictly mixed voting strategy, namely, they randomize between voting for and against the proposal. Next, we discuss the intuition behind Lemma 1 in detail.

First, there must exist informed shareholders in equilibrium, that is, $q \neq 0$, because otherwise, the PA could always adjust its selling strategy to induce information demand and to make positive profits. Moreover, the informed shareholders must follow the voting recommendation after purchasing it; that is, $v(1) = 1$ and $v(0) = 0$.[7] Otherwise, if the informed shareholders were to always vote in the same way regardless of the voting recommendations, they would be better off not buying the voting recommendation in the first place. Alter-

[7]By simple relabeling, PA can recommend $r = 0$ if p is large and $r = 1$ if p is small, and informed shareholders simply reverse their strategies; that is, they vote for the proposal if $r = 0$ and against it if $r = 1$. However, this equilibrium is trivial, and we rule it out.

natively, if the informed shareholders were to adopt a strictly mixed voting strategy when receiving $r = 1$ and/or when receiving $r = 0$, they would be indifferent between voting according to the recommendation and voting against it. Thus, again, they would be better off not buying the recommendation initially.

Informed shareholders must vote according to the advice of the PA, so $q < 1$ in equilibrium. Otherwise, if every shareholder were to buy the voting recommendation, that is, $q = 1$, as argued above, all shareholders would vote in the same way as recommended by the PA. In this case, it would be impossible for a shareholder to change the voting outcome, or equivalently, she could never be a *pivotal* shareholder (the number of "for" and "against" votes would be equal among other shareholders). Thus, the recommendation would be worthless and no shareholders would buy it. Therefore, there must be uninformed shareholders in equilibrium, that is, $q < 1$.

Second, in equilibrium the uninformed shareholders must use a strictly mixed voting strategy. Otherwise, they would suffer from the "swing voter's curse" (Feddersen and Pesendorfer, 1996). To explain this scenario, without loss of generality, assume toward a contradiction that all uninformed shareholders vote in favor of the proposal. Then because the informed shareholders follow their received voting recommendations, a randomly selected shareholder will vote for the proposal with a probability of 1 if the PA's recommendation is $r = 1$ and with a probability of $1 - q$ if the recommendation is $r = 0$. Hence, an uninformed shareholder can *possibly influence* the voting outcome if and only if $r = 0$. However, in such a scenario, the uninformed shareholder does not want to vote for the proposal, because she can infer that the informed shareholders' private signal is $r = 0$.

In fact, conditional on other shareholders' voting strategies, such an uninformed shareholder's vote matters only if she is pivotal. Each shareholder has a positive probability of being pivotal in equilibrium, so the shareholder casts her vote as if she is pivotal.[8] Interestingly, as analyzed above, the event of being pivotal per se is informative. That is, even though the uninformed shareholder does not observe the information directly from the PA,

[8] If she is not pivotal, her vote cannot change the voting outcome. Thus, she only considers the case in which she can make a difference.

she can infer that the voting recommendation is more likely to be $r = 0$ than $r = 1$ from the event of being pivotal. Thus, continuing the above analysis, she should lower her probability of voting for the proposal to reflect this information. Each uninformed shareholder performs this calculation and adjusts her voting strategy accordingly. In equilibrium, the uninformed shareholders' voting strategy is adjusted to the point at which being pivotal implies that voting for or against yields the same payoff for them.

A rejected proposal yields zero payoff, so the indifference condition for the uninformed shareholders implies that the expected payoff of voting for the proposal also should be zero. Mathematically, given shareholders' utility function (equation (2)), the indifference condition can be expressed as $\Pr(\theta = 1|PIV_i) - \gamma \Pr(\theta = 0|PIV_i) = 0$, where PIV_i denotes the event of being pivotal. After further simplification (see the appendix for the proof), we obtain the following:

$$\Pr(PIV_i|r = 1)A_1 - \Pr(PIV_i|r = 0)A_0 = 0, \tag{10}$$

where A_1 and A_0 are obtained via equations (6) and (7), respectively. Equation (10) expresses that, in equilibrium, the following two terms should be equal for the uninformed shareholder: (a) the payoff that is obtained if she infers from the event of being pivotal that the recommendation is $r = 1$ and she votes for the proposal and (b) the loss that is avoided if she infers from the event of being pivotal that the recommendation is $r = 0$ and she votes against the proposal. In addition, solving this indifference condition (10) yields the voting strategy of the uninformed shareholder, namely, $v(\varnothing) = \tau$, where τ is calculated via equation (8).

Interestingly, with equation (10), M, which is defined in equation (9) and depends on the ratio of A_0 to A_1, has an alternative interpretation:

$$M = M(\varrho) = \sqrt[\frac{N-1}{2}]{\frac{A_0}{A_1}} = \sqrt[\frac{N-1}{2}]{\frac{\Pr(PIV_i|r = 1)}{\Pr(PIV_i|r = 0)}}. \tag{11}$$

Thus, M also captures the probability of being pivotal when the PA issues $r = 1$ relative to that when it issues $r = 0$.

2.2 Information-purchase-stage equilibrium

At the information-purchase stage, given the PA's selling strategy (f, ϱ), shareholders make information-purchase decisions that maximize their expected utility net of the potential subscription fee for the recommendation. Lemma 2 summarizes the equilibrium at the information-purchase stage.

Lemma 2 (Equilibrium information-purchase strategy)**.** *At $t = 1$ (the information-purchase stage), (a) if $f > \tilde{f}$, where $\tilde{f}$ is given by equation (A18) in the appendix, $q(f, \varrho) = 0$; (b) if $f = 0$, $q(f, \varrho) = 1$; and (c) if $0 < f \leq \tilde{f}$,*

$$q(f, \varrho) = \sqrt{\frac{M - 2G(M+1) + \sqrt{M(1-4G)(M-4G)}}{2M}} \in (0, 1), \tag{12}$$

where $M = M(\varrho)$ is given by equation (9), and G is defined as a function of f and ϱ as follows:

$$G = G(f, \varrho) = \sqrt[\frac{N-1}{2}]{\frac{f}{\binom{N-1}{\frac{N-1}{2}} A_1}}, \tag{13}$$

where $\binom{N-1}{\frac{N-1}{2}} = \frac{(N-1)!}{(\frac{N-1}{2})!(\frac{N-1}{2})!}$ and A_1 is given by equation (6).

According to Lemma 2, given the recommendation threshold ϱ, if the price of the recommendation is too prohibitive $(f > \tilde{f})$, no shareholders buy it, and if it is free $(f = 0)$, every shareholder buys it. Only for a moderate price $(0 < f \leq \tilde{f})$ does the information value equal its price; thus, shareholders use a strictly mixed information-purchase strategy, that is, $q \in (0, 1)$. According to Lemma 1, only $q \in (0, 1)$ lies on the equilibrium path. Moreover, for a recommendation threshold, the higher the information price f, the lower the information demand q.

If $0 \leq f < \tilde{f}$, there exist two equilibria; that is, a shareholder can adopt two distinct information-purchase strategies under the same information price. Following Malenko and Malenko (2019), we assume that if multiple equilibria exist at the information-purchase stage, shareholders coordinate on the equilibrium in which shareholder welfare is maximized.[9]

[9] See Assumption A3 in the proof of Lemma 1 in the appendix for a detailed discussion.

Therefore, the recommendation threshold ϱ is fixed, so a unique information demand q corresponds to any specified price $f \geq 0$, as characterized in Lemma 2.

2.3 Profit maximization stage

With the well-defined demand function, $q(f, \varrho)$, for any specified ϱ, the PA can determine the information demand q directly by announcing the information price f. Now, the PA's problem, which was summarized in equation (5), becomes the selection of (q, ϱ), rather than (f, ϱ), to maximize the profits from proxy voting services and any other potential conflicted services as follows:

$$\max_{q,\varrho} q\, f(q, \varrho) + \Pr(r = 1)\, \varphi. \tag{14}$$

From this point on, we consider the PA's maximization problem that is specified in equation (14). Once we have solved for the PA's optimal selling strategy (q^*, ϱ^*), the optimal price f^* can be uniquely determined. The following proposition characterizes the PA's optimal selling strategy (q^*, ϱ^*).

Proposition 1 (Optimal selling strategy). *In equilibrium, there exists a unique (q^*, ϱ^*) that maximizes the PA's profits. Specifically, if $\varphi = 0$, $\varrho^* = \hat{\varrho}$, where $\hat{\varrho}$ satisfies the following equation:*

$$\frac{h_1(\hat{\varrho})}{h_0(\hat{\varrho})} = \frac{\gamma(1-\mu)}{\mu}, \tag{15}$$

and if $\varphi > (<)\, 0$, $\varrho^* < (>)\, \hat{\varrho}$. For any φ, $q^* = q^*(\varrho^*)$, where $q^*(\cdot)$ is given by the following equation:

$$q^*(\varrho) = \frac{\sqrt{4M - N(1-M)^2 + (1+M)\sqrt{4M + N^2(1-M)^2}}}{2\sqrt{2NM}}. \tag{16}$$

where $M = M(\varrho)$ is given by equation (9).

Proposition 1 states that the optimal recommendation threshold ϱ^* depends on whether the PA is conflicted. If the PA is not conflicted, the equilibrium threshold of sending positive recommendations is uniquely determined by equation (15), which only depends on shareholders' state-contingent payoffs γ and their subjective beliefs μ. However, if the PA is conflicted, its voting recommendation is contaminated by the conflicts: if by issuing positive recommendations the PA obtains additional payoff, it is more likely to recommend approving the proposal compared with the case without conflicts, that is, $\varrho^* < \hat{\varrho}$ if $\varphi > 0$. Instead, if the PA incurs a loss by recommending approval of the proposal, it will be less likely to do so, that is, $\varrho^* > \hat{\varrho}$ if $\varphi < 0$.

Regardless of whether the PA is conflicted, once the optimal threshold ϱ^* has been set, the optimal information demand q^* is solved via equation (16). In other words, the PA's profit maximization can be realized in two steps: designing its information product (characterized by ϱ^*) and, given the product, determining the optimal information demand/supply by announcing the information price (characterized by q^*).

Further, Proposition 1 implies that the PA's profits from proxy voting services are maximized at $\varrho = \hat{\varrho}$, whereas the information demand q^* is minimized at that point. Because the monopolistic PA's profits from proxy voting services are equal to the product of the information demand and the information value,[10] it suggests that at $\varrho = \hat{\varrho}$, the information value is maximized. Therefore, when the PA is designing its recommendation threshold, namely, contemplating whether to send $r = 1$ or $r = 0$ more often, the consideration of the effect of ϱ on the information value takes priority. In other words, if faced with a trade-off between margins and volumes, the PA adjusts the recommendation threshold to maximize the margin of its information product. Overall, the PA's optimal selling strategy pursues "high margins with low volumes." This is because in a strategic voting game, every shareholder has an incentive to free ride on others' informed votes. As more shareholders become informed, the marginal value of the information substantially depreciates, as do shareholders' incentives to purchase the recommendation. Therefore, the PA tries to minimize the number of informed shareholders to preserve the information value and to maximize the profits from the proxy voting business accordingly.

[10]By virtue of being a monopolist, the PA can extract all information rents by setting the price to the information value.

3 Information Bias in the Proxy Advisory Market

In this section, we define the unbiased voting recommendation. Then we analyze a benchmark economy that is free from all potential sources of bias. Finally, we explore the primary result of the paper: buy-side imperfections can be passed on to proxy voting recommendations, but some biased recommendations may induce more shareholders to purchase recommendations and cast informed votes, thereby increasing firm value.

3.1 Unbiased recommendations and firm value

Definition 2 (Unbiased recommendations)**.** *A voting recommendation is unbiased if and only if* $r = \mathbb{I}\{\Pr(\theta = 1|p, \mu_0) \geq \Pr(\theta = 0|p, \mu_0)\}$*, where* $\mathbb{I}\{\cdot\}$ *is the indicator function.*

By definition, in the presence of evidence p, if the posterior probability of the state being good $\Pr(\theta = 1|p, \mu_0)$ is higher (lower) than that of the state being bad $\Pr(\theta = 0|p, \mu_0)$, an unbiased voting recommendation should advise shareholders to vote for (against) the proposal. In other words, a voting recommendation is unbiased if and only if it is consistent with the more likely state given the evidence. According to this definition, one can easily verify that the PA's recommendation is unbiased if and only if it issues a positive recommendation if $p \geq \varrho^{\mathrm{B}}$, where ϱ^{B} is the unique solution to the following equation (see the appendix for a formal proof):[11]

$$\frac{h_1(\varrho^{\mathrm{B}})}{h_0(\varrho^{\mathrm{B}})} = \frac{1-\mu_0}{\mu_0}. \tag{17}$$

Note that the threshold for the unbiased voting recommendation depends only on the true prior μ_0 and not on shareholders' subjective prior μ.

For welfare analysis, we focus on firm value. Each shareholder owns one share, so the expected firm value is equal to the number of shareholders (N) times each share value. Thus, we simply refer to the per capita form as firm value, which is denoted by Ψ and computed as $\Psi = \Pr(\theta = 1, d = 1)\psi(1, 1) + \Pr(\theta = 0, d = 1)\psi(0, 1)$, where $\psi(\theta, d)$ is specified by

[11] As will be clarified in Section 3.3, we use the superscript B to represent unbiased recommendation, because it is the equilibrium recommendation threshold in the benchmark economy.

equation (1). Firm value depends on the PA's choice of q and ϱ under the parameters γ, μ and φ, and can be further simplified as follows:

$$\Psi(q, \varrho; \gamma, \mu, \varphi) = \mu_0 \Pr(d = 1|\theta = 1) - (1 - \mu_0) \Pr(d = 1|\theta = 0). \tag{18}$$

3.2 Single-shareholder case

Before analyzing the equilibrium with the complete voting game, it is useful to consider two hypothetical scenarios in which the voting friction is absent.

First, we consider the case in which the implementation of the proposal is solely based on the PA's recommendation. That is, if the PA recommends voting for (against) the proposal, then the proposal is accepted (rejected). In this hypothetical case, what is the recommendation that maximizes the expected firm value? The following lemma summarizes our finding.

Lemma 3. *If the implementation of the proposal is solely based on the PA's recommendation, the maximal firm value is realized by following the unbiased voting recommendation.*

Lemma 3 states that the decision policy following the unbiased voting recommendation maximizes the expected firm value. In other words, if the proposal passes (fails) when the unbiased recommendation is to vote yes (no), the expected firm value reaches the highest level, which we denote by Ψ^0. This represents the most efficient use of the PA's private information.

Next, we consider the case in which the firm is owned by a single shareholder. Compared with the previous scenario, 1 of 3 types of frictions can occur: (1) the PA has conflicts of interest; (2) shareholders do not aim to maximize firm value; or (3) shareholders hold incorrect subjective prior beliefs. In addition, the only difference between this setting and the main model is that voting friction does not occur.

Lemma 4. *Suppose the firm is owned only by one shareholder with state-contingent payoff γ and subjective belief μ.*

(i) In equilibrium, the shareholder purchases the PA's recommendation with probability 1 and follows the received recommendation to pass or fail the proposal.

(ii) *If $\varphi = 0$, the PA sets the recommendation threshold to $\hat{\varrho}$, where $\hat{\varrho}$ is specified by equation* (15).

(iii) *If $\varphi = 0, \gamma = 1$, and $\mu = \mu_0$, the PA sends the unbiased voting recommendation $(\varrho = \varrho^{\mathrm{B}})$ and firm value reaches Ψ^0, where Ψ^0 is firm value in Lemma 3.*

Lemma 4 states that in a single-shareholder case, the shareholder would fully delegate her voting decision to the PA (part (i) of this lemma), and the PA, if not conflicted, would act as if it were the firm owner (part (ii) of this lemma). Compared with Lemma 3, according to part (iii) of Lemma 4, having a monopolistic PA per se is not a sufficient condition for the potential inefficiency: the highest expected firm value still can be realized. This is because without the free-riding incentive in the voting game, the single shareholder must bear the total cost of the decision making and considers all available information.

The common theme in Lemmas 3 and 4 is that the unbiased voting recommendation maximizes firm value if there is no voting friction, which, again, helps support the justification of the unbiased voting recommendation as an important criterion for assessing information quality in the proxy advisory market.

3.3 Benchmark

Now we are ready to discuss information bias in the proxy advisory market. Initially, we consider a case without potential sources of bias, namely, the PA does not have conflicts of interest, shareholders aim to maximize firm value, and shareholders hold correct beliefs ($\varphi = 0$, $\gamma = 1$, and $\mu = \mu_0$). This serves as a benchmark for exploration of information bias in the market and its welfare implications. The PA's optimal selling strategy is designated by $(q^{\mathrm{B}}, \varrho^{\mathrm{B}})$, where the superscript B denotes the benchmark economy. The following proposition summarizes the equilibrium.

Proposition 2 (Benchmark)**.** *Assume $\varphi = 0, \gamma = 1$, and $\mu = \mu_0$. In equilibrium, the PA sends unbiased voting recommendations ϱ^{B}, and the information demand is $q^{\mathrm{B}} = q^*(\varrho^{\mathrm{B}})$, where $q^*(\cdot)$ is specified by equation* (16) *under the parameters $\gamma = 1$ and $\mu = \mu_0$. Given q^{B}, firm value is maximized with respect to ϱ at $\varrho = \varrho^{\mathrm{B}}$.*

The equilibrium firm value in this benchmark is designated by Ψ^{B}, that is, $\Psi^{\mathrm{B}} \equiv \Psi(q^{\mathrm{B}}, \varrho^{\mathrm{B}}; 1, \mu_0, 0)$. In the benchmark, the PA sends unbiased recommendations to shareholders; that is, the PA recommends approving the proposal only if it thinks the state is more likely to be good. Given the equilibrium information demand, firm value is maximized with respect to the threshold at $\varrho = \varrho^{\mathrm{B}}$. In other words, holding q^{B} fixed, firm value cannot be increased by adjusting the PA's recommendation threshold. Comparing this firm value with the value in the hypothetical case that is defined by Lemma 3, we obtain the following result.

Corollary 1. *The equilibrium firm value in the benchmark is lower than that in the hypothetical case in which the implementation of the proposal is solely based on the unbiased recommendation, namely, $\Psi^{\mathrm{B}} < \Psi^0$, where Ψ^0 and Ψ^{B} are firm values in Lemma 3 and Proposition 2, respectively.*

This reduction of firm value in the benchmark relative to that in the hypothetical case, as stated in Corollary 1, arises from not every shareholder subscribing to the recommendation in the benchmark, namely, $q^{\mathrm{B}} < 1$. This is due to two frictions: the voting friction, that is, every shareholder has an incentive to free ride, and the PA's monopolistic pricing. In the extreme case, if all shareholders buy the recommendation with probability 1, this advice has no value to a marginal shareholder and, thus, she is not willing to pay for it. Therefore, to preserve the value of its recommendation and, thus, to maximize profits, the PA rations the supply of its recommendation, thereby leaving room for firm value increase if q can increase over q^{B}. Overall, the comparison in Corollary 1 highlights the difference between the single-decision problem and the collective-decision problem.

3.4 Supply-driven bias

Next, we examine the supply-driven force that can affect the information quality in the proxy advisory market. We consider the effect of the PA's conflicts of interest, which is summarized in Corollary 2.

Corollary 2 (Supply-driven bias)**.** *Assume that shareholders aim to maximize firm value and hold correct beliefs but the PA has conflicts of interest ($\gamma = 1$ and $\mu = \mu_0$, but $\varphi \neq 0$). The*

PA's voting recommendation is biased, that is, $\varrho^* < (>)\, \varrho^{\mathrm{B}}$ if $\varphi > (<)\, 0$. There exists a number $\varepsilon_\varphi > 0$ such that if $|\varphi| < \varepsilon_\varphi$, compared with the benchmark, the information demand is higher, that is, $q^* \geq q^{\mathrm{B}}$; and if $\mu_0 = 1/2$, firm value is lower, that is, $\Psi(q^*, \varrho^*; 1, \mu_0, \varphi) < \Psi^{\mathrm{B}}$.

All else equal, if there are conflicts of interest on the PA side, according to Corollary 2, the PA's voting recommendations are biased. It is intuitive that the PA issues more positive (negative) recommendations if it earns extra payoff by recommending approval (disapproval) of the proposal; that is, if $\varphi > 0$, $\varrho^* < \varrho^{\mathrm{B}}$, and if $\varphi < 0$, $\varrho^* > \varrho^{\mathrm{B}}$.

Next, according to Proposition 1, φ affects q^* through ϱ^* only. As the PA's conflicts of interest drive the equilibrium recommendation threshold ϱ^* away from ϱ^{B}, the recommendation is of less value but it has a higher volume of sales, that is, q^* increases.

Last, as stated in Corollary 2, the presence of any supply-driven bias reduces firm value. Intuitively, because shareholders endeavor to maximize firm value, the PA's recommendation is valued the most (and, hence, results in the largest profits) if it helps realize that objective. This occurs if the PA is unconflicted. The presence of any conflicts of interest on the PA side only reduces firm value.[12] Although more shareholders purchase the voting recommendation and, thus, cast informed (but biased) votes, this potential positive effect is always overshadowed by the negative consequences of the information bias (which will be discussed in detail later). Hence, the supply-driven bias unambiguously decreases firm value.

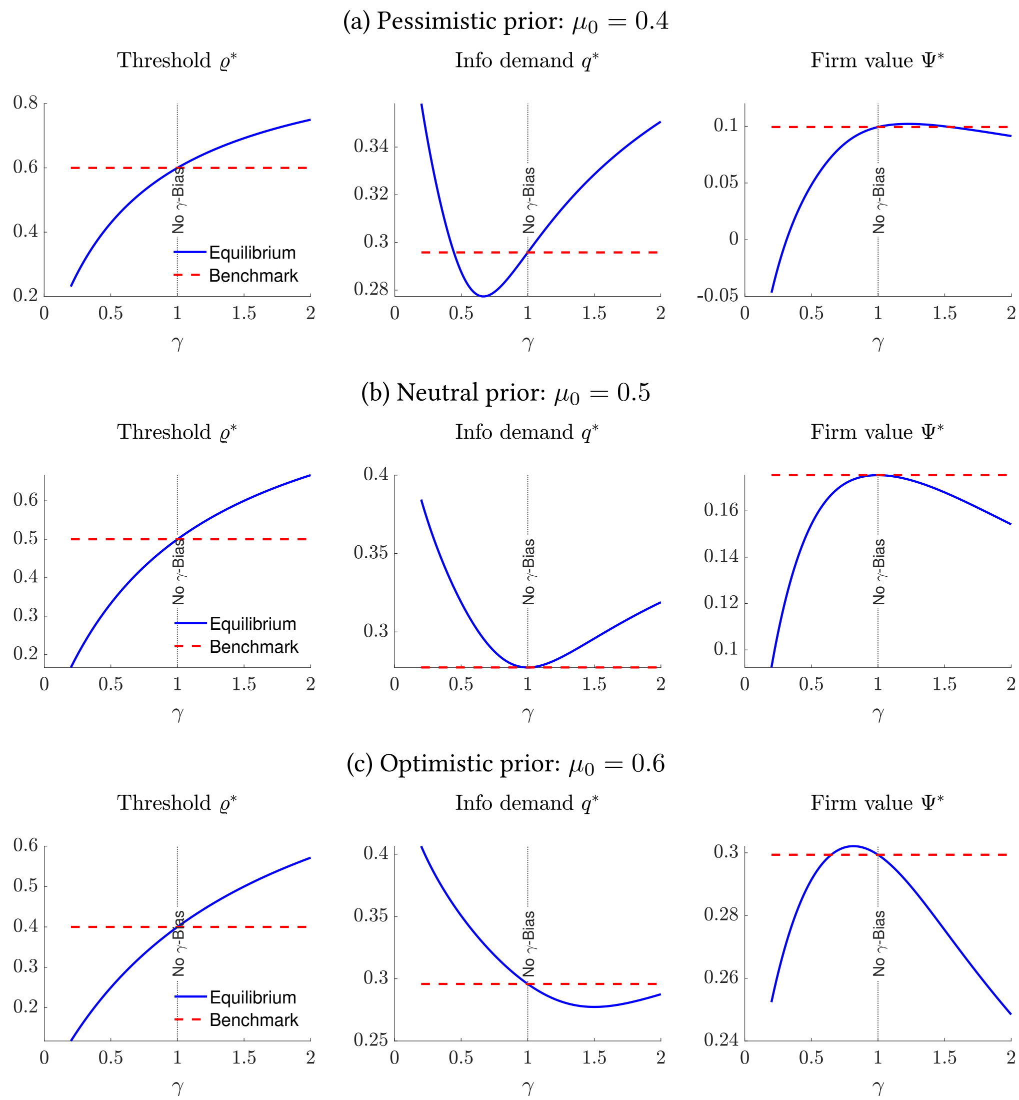
3.5 Demand-driven bias

We now shut down the PA's conflicts of interest and focus on the bias that arises from the demand side, that is, shareholders' preferences γ and their subjective beliefs μ. We refer to the former as the γ-bias and the latter as the μ-bias. Propositions 3 and 4 summarize the effects of the γ- and μ-biases, respectively, and Figures 2 and 3 graphically present Propositions 3 and 4, respectively.

Proposition 3 (Bias from shareholders' preferences)**.** *Assume that the PA has no conflicts of interest, and shareholders hold correct prior beliefs but do not maximize firm value ($\varphi = 0$,*

[12]We have proved this for the case in which $\mu_0 = \frac{1}{2}$, and numerical examples indicate the result holds for other values of μ_0 as well.

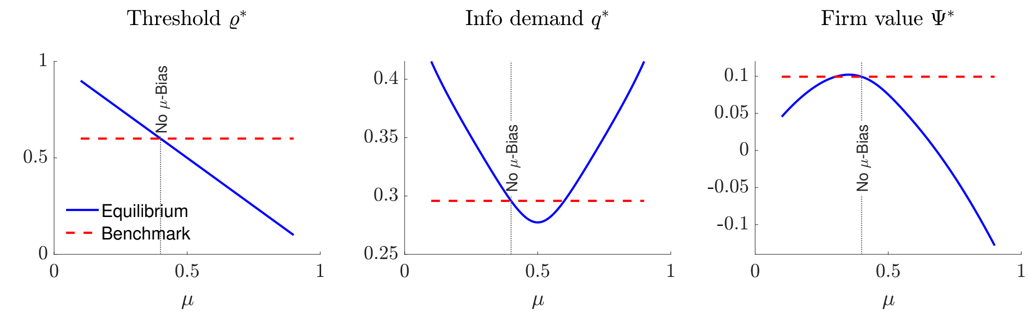
Figure 2: Effect of shareholders' preference γ



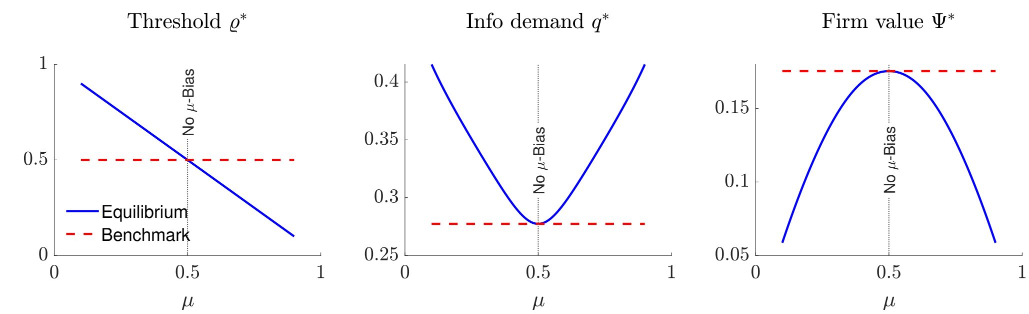
The figure plots the effect of shareholders' loss aversion parameter γ on the threshold of sending positive recommendations ϱ^*, information demand q^*, and firm value Ψ^*. Panels A–C plot the economy with true prior to be pessimistic ($\mu_0 = 0.4$), neutral ($\mu_0 = 0.5$), and optimistic ($\mu_0 = 0.6$), respectively. Parameters used are as follows: $N = 13$, $\varphi = 0$, and $\mu = \mu_0$. The conditional densities of the PA's private signal p are specified as $h_1(p) = 2p$ and $h_0(p) = 2(1-p)$. The vertical line represents the case in which $\gamma = 1$; that is, shareholders aim to maximize firm value.

Figure 3: Effect of shareholders' subjective beliefs μ

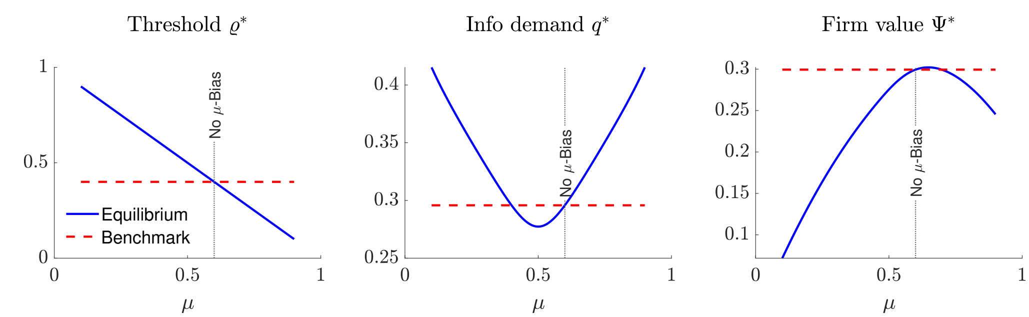
(a) Pessimistic prior: $\mu_0 = 0.4$



(b) Neutral prior: $\mu_0 = 0.5$



(c) Optimistic prior: $\mu_0 = 0.6$



The figure plots the effect of shareholders' subjective prior μ on the threshold of sending positive recommendations ϱ^*, information demand q^*, and firm value Ψ^*. Panels A–C plot the economy with true prior to be pessimistic ($\mu_0 = 0.4$), neutral ($\mu_0 = 0.5$), and optimistic ($\mu_0 = 0.6$), respectively. Parameters are as follows: $N = 13$, $\varphi = 0$, and $\gamma = 1$. The specification of the conditional densities of the PA's private signal are the same as in Figure 2. The vertical line represents the case in which $\mu = \mu_0$, that is, the prior beliefs of shareholders are correct.

$\mu = \mu_0$, *but* $\gamma \neq 1$). *The PA's recommendation is biased such that* $\varrho^* > (<)\, \varrho^{\mathrm{B}}$ *if* $\gamma > (<)\, 1$. *There exists some number* $\varepsilon_\gamma > 0$ *such that when* $|\gamma - 1| < \varepsilon_\gamma$, *compared with the benchmark, we have the following results:*

1. *if shareholders would have voted against the proposal based on their prior* ($\gamma > \frac{\mu}{1-\mu}$, *or equivalently* $\mu_0 < \frac{1}{2}$), *when* $1 < \gamma < 1 + \varepsilon_\gamma$ ($1 - \varepsilon_\gamma < \gamma < 1$), *the information demand is higher* (*lower*), *that is,* $q^* > (<)\, q^{\mathrm{B}}$, *and firm value is higher* (*lower*), *that is,* $\Psi(q^*, \varrho^*; \gamma, \mu_0, 0) > (<)\, \Psi^{\mathrm{B}}$;
2. *if shareholders would have voted for the proposal based on their prior* ($\gamma < \frac{\mu}{1-\mu}$, *or equivalently* $\mu_0 > \frac{1}{2}$), *when* $1 < \gamma < 1 + \varepsilon_\gamma$ ($1 - \varepsilon_\gamma < \gamma < 1$), *the information demand is lower* (*higher*), *that is,* $q^* < (>)\, q^{\mathrm{B}}$, *and firm value is lower* (*higher*), *that is,* $\Psi(q^*, \varrho^*; \gamma, \mu_0, 0) < (>)\, \Psi^{\mathrm{B}}$; *and*
3. *if shareholders would have been indifferent to voting for or against the proposal based on their prior* ($\gamma = \frac{\mu}{1-\mu}$, *or equivalently* $\mu_0 = \frac{1}{2}$), *the information demand is always higher, that is,* $q^* > q^{\mathrm{B}}$, *and firm value is always lower, that is,* $\Psi(q^*, \varrho^*; \gamma, \mu_0, 0) < \Psi^{\mathrm{B}}$.

Proposition 4 (Bias from shareholders' prior beliefs)**.** *Assume that the PA has no conflicts of interest, and shareholders maximize firm value but do not hold correct prior beliefs* ($\varphi = 0$, $\gamma = 1$, *but* $\mu \neq \mu_0$). *The PA's recommendation is biased such that* $\varrho^* < (>)\, \varrho^{\mathrm{B}}$ *if* $\mu > (<)\, \mu_0$. *There exists some number* $\varepsilon_\mu > 0$ *such that when* $|\mu - \mu_0| < \varepsilon_\mu$, *compared with the benchmark, we have the following results:*

1. *if shareholders would have voted against the proposal based on their prior* ($\gamma > \frac{\mu}{1-\mu}$, *or equivalently* $\mu_0 < \frac{1}{2}$), *when* $\mu_0 < \mu < \mu_0 + \varepsilon_\mu$ ($\mu_0 - \varepsilon_\mu < \mu < \mu_0$), *the information demand is higher* (*lower*), *that is,* $q^* < (>)\, q^{\mathrm{B}}$, *and firm value is lower (higher), that is,* $\Psi(q^*, \varrho^*; 1, \mu, 0) < (>)\, \Psi^{\mathrm{B}}$;
2. *if shareholders would have voted for the proposal based on their prior* ($\gamma < \frac{\mu}{1-\mu}$, *or equivalently* $\mu_0 > \frac{1}{2}$), *when* $\mu_0 < \mu < \mu_0 + \varepsilon_\mu$ ($\mu_0 - \varepsilon_\mu < \mu < \mu_0$), *the information demand is higher* (*lower*), *that is,* $q^* > (<)\, q^{\mathrm{B}}$, *and firm value is higher* (*lower*), *that is,* $\Psi(q^*, \varrho^*; 1, \mu, 0) > (<)\, \Psi^{\mathrm{B}}$; *and*

3. *if shareholders would have been indifferent to voting for or against the proposal based on their prior* ($\gamma = \frac{\mu}{1-\mu}$, *or equivalently* $\mu_0 = \frac{1}{2}$), *the information demand is always higher, that is,* $q^* > q^{\mathrm{B}}$, *and firm value is always lower, that is,* $\Psi(q^*, \varrho^*; 1, \mu, 0) < \Psi^{\mathrm{B}}$.

First, Propositions 3 and 4 state that the PA skews its voting recommendations based on its clients' preferences or beliefs. If shareholders lose more upon false acceptance of a bad proposal ($\gamma > 1$) or if shareholders are overly pessimistic ($\mu < \mu_0$), the PA is biased toward issuing a negative recommendation, that is, $\varrho^* > \varrho^{\mathrm{B}}$; otherwise, if shareholders are more tolerant of a falsely accepted proposal ($\gamma < 1$) or if they are overly optimistic ($\mu > \mu_0$), the PA tends to send positive voting recommendations, that is, $\varrho^* < \varrho^{\mathrm{B}}$.

Why doesn't the unconflicted PA issue unbiased voting recommendations? Recall that in Lemma 1, the equilibrium voting and information-purchase strategies depend on μ (not μ_0) and γ. In other words, shareholders use their preferences and subjective prior beliefs to evaluate their voting and information-purchase strategies. Consequently, knowing how its recommendations affect its clients' subsequent information purchase and voting, the PA only considers shareholders' subjective prior belief parameter μ and their preference parameter γ. Therefore, even without any conflicts of interest, the PA does not necessarily issue unbiased recommendations. The following corollary summarizes this result.

Corollary 3 (Shareholders' subjective utility)**.** *Assume that the PA has no conflicts of interest* ($\varphi = 0$)*. The equilibrium recommendation threshold* ϱ^* *maximizes shareholders' subjective expected utility.*

Second, according to Proposition 3, in the presence of demand-driven bias, firm value can exceed that in the benchmark in some cases. How can a *biased* voting recommendation *increase* firm value relative to that in the benchmark economy in which each and every shareholder holds the correct prior beliefs and aims at *maximizing* firm value? Also, recall that in Lemmas 3 and 4, firm value is maximized if and only if the implementation of the proposal is consistent with the unbiased recommendation. This renders the increase in firm value that is observed here all the more striking.

Firm value is determined by the fraction of informed shareholders (q^*) and the quality of the recommendation that each of them uses to vote (ϱ^*). The demand-driven bias affects firm value via these two channels. Now, we consider the γ-bias as an example and study the limiting case in which $\gamma \to 1$. We discuss why a small deviation of γ from the benchmark ($\gamma = 1$) can improve firm value in some cases. Throughout this discussion, we assume that $\mu = \mu_0$; that is, shareholders' beliefs over states are correct.

Calculating the total derivative of Ψ with respect to γ yields the following:

$$\frac{\mathrm{d}\Psi}{\mathrm{d}\gamma} = \underbrace{\frac{\partial \Psi}{\partial \gamma}}_{=0 \text{ at } \varrho=\varrho^{\mathrm{B}}} + \underbrace{\frac{\partial \Psi}{\partial \varrho}}_{=0 \text{ at } \varrho=\varrho^{\mathrm{B}}} \cdot \frac{\mathrm{d}\varrho}{\mathrm{d}\gamma} + \frac{\partial \Psi}{\partial q} \cdot \underbrace{\frac{\mathrm{d}q}{\mathrm{d}\gamma}}_{\substack{\text{may not be } 0 \\ \text{at } \varrho=\varrho^{\mathrm{B}}}}. \tag{19}$$

According to equation (19), the γ-bias affects firm value directly ($\frac{\partial \Psi}{\partial \gamma}$) and indirectly through the information acquisition q^* ($\frac{\partial \Psi}{\partial q}\frac{\mathrm{d}q}{\mathrm{d}\gamma}$) and the information quality ($\frac{\partial \Psi}{\partial \varrho}\frac{\mathrm{d}\varrho}{\mathrm{d}\gamma}$). First, the direct effect captures the deviation of shareholders' preferences from firm-value maximization. Intuitively, the more γ deviates from 1, the less shareholders care about firm value and the smaller the equilibrium firm value. However, that the marginal direct effect of the γ-bias is zero at $\varrho = \varrho^{\mathrm{B}}$ does not explain how the increase in firm value is associated with biased recommendations . Second, according to Proposition 1, firm value is maximized with respect to ϱ at $\varrho = \varrho^{\mathrm{B}}$. Any deviation from ϱ^{B} can only decrease firm value. Thus, again, this is not the reason firm value can increase. Third, at $\varrho = \varrho^{\mathrm{B}}$, the marginal change of q in response to γ is not necessarily zero, namely, $\frac{\mathrm{d}q}{\mathrm{d}\gamma} \neq 0$ in general.

Although the first two channels reduce firm value, the last one increases it. We refer to these novel effects on firm value as the bias-quantity relationship. If the quantity effect prevails, firm value may increase in the presence of information bias. According to equation (19), in the limiting case in which $\gamma \to 1$, the bias effects are muted ($\frac{\partial \Psi}{\partial \gamma} = 0$ and $\frac{\partial \Psi}{\partial \varrho}\frac{\mathrm{d}\varrho}{\mathrm{d}\gamma} = 0$) and the quantity effect, if it exists, dominates and determines firm value.

Next, we examine the quantity effect in detail. Let us first consider the case in which $\mu_0 < \frac{1}{2}$ (the analysis of the case in which $\mu_0 > \frac{1}{2}$ is similar). If $\mu_0 < \frac{1}{2}$, a shareholder will vote against the proposal if she has only her prior to consider. Based on Corollary 1, firm value in the benchmark is lower than the highest level Ψ_0 due to two frictions: (1) the free-riding problem in the voting game and (2) the monopolistic PA rationing the supply of its recommendations. Now, suppose that shareholders become more loss averse, that is, $1 < \gamma < 1 + \varepsilon_\gamma$. With the introduction of this buy-side bias, shareholders' free-riding incentives have increased. This is because as their prior conservative voting tendency is strengthened, namely, they become more willing to vote against the proposal, the votes are less likely to be split, and the likelihood of being pivotal decreases. This ultimately depresses shareholders' incentives to purchase the recommendation and lowers the information value. In response, the monopolistic PA increases the supply of recommendations to maximize its profits, that is, q^* increases. In essence, the interesting interplay between the voting friction and the monopolistic PA's optimization problem increases firm value.

In summary, while the for-profit PA skews its recommendation based on shareholders' preferences, in some cases such catering can be associated with more information acquisition in equilibrium. The finding that q^* can increase as the result of biased recommendations is the fundamental reason firm value may increase in the presence of the γ-bias. As more shareholders cast their votes in an informed way, or at least not solely based on their priors and the equilibrium information (the information regarding being pivotal), the quality of their collective decision-making can be enhanced.

To gauge the effect of the γ-bias on q^* mathematically, we calculate the total derivative of q^* and obtain the followings:

$$\frac{\mathrm{d}q}{\mathrm{d}\gamma} = \frac{\partial q}{\partial M}\Big(\underbrace{\frac{\partial M}{\partial \varrho}}_{=0} \cdot \frac{\mathrm{d}\varrho}{\mathrm{d}\gamma} + \underbrace{\frac{\partial M}{\partial \gamma}}_{>0}\Big) = \frac{\partial q}{\partial M} \cdot \frac{\partial M}{\partial \gamma}. \tag{20}$$

The γ-bias clearly affects the information acquisition q^*, both indirectly through ϱ $(\frac{\partial M}{\partial \varrho}\frac{\mathrm{d}\varrho}{\mathrm{d}\gamma})$ and directly $(\frac{\partial M}{\partial \gamma})$. Because in equilibrium, the PA cannot further increase the profits by adjusting the frequency of sending positive and negative recommendations, we have $\frac{\partial M}{\partial \varrho} = 0$. Furthermore, Proposition 1 implies that as A_0 and A_1 diverge (M moves away from 1), q^* increases. In the case of $\mu_0 < \frac{1}{2}$, we have $A_0 > A_1$ so that $M > 1$. As γ increases, M is pushed further larger than 1; hence, q^* increases $(\frac{\partial q}{\partial M} > 0)$. In summary, we have shown that if $\mu_0 < \frac{1}{2}$ and $1 < \gamma < 1 + \varepsilon_\gamma$, the equilibrium information acquisition can increase

$(\frac{\mathrm{d}q}{\mathrm{d}\gamma} > 0)$ and the consequent firm value can exceed that in the benchmark, as plotted in panel A of Figure 2.

Now we consider the case in which $\mu_0 = \frac{1}{2}$. If $\mu_0 = \frac{1}{2}$, shareholders do not have any prior voting tendency, namely, they randomize between voting for and against the proposal according to their priors. As argued above, firm value in the benchmark falls short of Ψ^0 in Lemmas 3 and 4 because of the voting friction and the PA's monopolistic pricing. Only if the quantity effect dominates the bias effect can firm value exceed that in the benchmark. However, in this case, the votes are highly likely to be split, and the probability of being pivotal is large, thereby rendering the recommendation highly valued by shareholders. Faced with such a high information value, the monopolistic PA can afford to ration the recommendation supply at the lowest level, that is, $q^* = \frac{1}{\sqrt{N}}$, thereby minimizing the negative effect of shareholders' free-riding incentives on its profits.[13] Therefore, in the limiting case in which $\gamma \to 1$, there does not exist any "quantity effect" $(\frac{\mathrm{d}q}{\mathrm{d}\gamma} = 0)$ and firm value cannot be further increased.

Finally, a natural question arises: for the supply-driven bias, why does the "quantity effect" never prevail? As we did for the demand-driven bias, we decompose the determinants of firm value and the quantity effect as follows:

$$\frac{\mathrm{d}\Psi}{\mathrm{d}\varphi} = \underbrace{\frac{\partial \Psi}{\partial \varrho}}_{=0 \text{ at } \varrho = \varrho^{\mathrm{B}}} \cdot \frac{\mathrm{d}\varrho}{\mathrm{d}\varphi} + \frac{\partial \Psi}{\partial q} \cdot \underbrace{\frac{\mathrm{d}q}{\mathrm{d}\varphi}}_{=0 \text{ at } \varrho = \varrho^{\mathrm{B}}}, \tag{21}$$

$$\frac{\mathrm{d}q}{\mathrm{d}\varphi} = \frac{\partial q}{\partial M} \cdot \underbrace{\frac{\partial M}{\partial \varrho}}_{=0 \text{ at } \varrho = \varrho^{\mathrm{B}}} \cdot \frac{\mathrm{d}\varrho}{\mathrm{d}\gamma}. \tag{22}$$

Comparing equation (21) to equation (19), for the supply-driven bias, in the limiting case around the benchmark, the "quantity effect" vanishes $(\frac{\mathrm{d}q}{\mathrm{d}\varphi} = 0)$. That this is the only force that can increase firm value suggests that the equilibrium firm value can never exceed that in the benchmark. A further comparison of equations (20) and (22) shows that the supply-driven bias affects q^* only indirectly. The indirect effect of the φ-bias on q^* is always muted

[13] One can easily show that $q^* = \frac{1}{\sqrt{N}}$ is less than any $q^*(\varrho)$ in equation (16).

at $\varrho = \varrho^{\mathrm{B}}$ (due to $\frac{\partial M}{\partial \varrho} = 0$), so the "quantity effect" can never occur and firm value never increases under the supply-driven bias.

In summary, while both supply and demand forces can introduce biases into the PA's voting recommendations, their distinct sources underlie their divergent implications for firm value: in the presence of conflicted businesses, the PA deliberately distorts the quality of its recommendation, whereas if faced with buy-side imperfections, the monopolistic PA optimally skews its recommendations in response. In the latter case, an interesting interplay occurs between the voting friction and the PA's monopolistic pricing, and the buy-side biases reinforce the voting friction, which partially undoes the inefficiency that is related to the PA's monopolistic information sales, thereby leading to increase in firm value.

4 Concluding Remarks

In this paper, we provide a framework for the study of information bias in the proxy advisory market. We find that only if (a) the proxy advisor has no conflicts of interest, (b) shareholders hold the correct prior beliefs, and (c) shareholders aim to maximize firm value does the proxy advisor provide unbiased voting recommendations. This holds even if the proxy advisor has no actual stake in the firm or there is no suitable oversight. In addition, we identify some potential limitations of the proxy advisor in enhancing informed corporate voting: as a for-profit information intermediary, the proxy advisor only caters to shareholders' subjective beliefs (even though it knows that such beliefs are incorrect) and their preferences (without regard for firm value), but it does not care about firm value. However, in some cases the proxy advisor's catering tendency can induce more shareholders to purchase the voting recommendations and cast informed votes, thereby increasing firm value.

The widespread use of proxy advisors has attracted substantial attention from regulators, and our framework helps inform the policy. For example, in a 2010 Concept Release on the U.S. Proxy System, the Securities and Exchange Commission (2010), identified various potential issues, such as the following:

1. if and to what extent proxy advisors develop, disseminate, and implement their voting

recommendations without adequate accountability for informational accuracy in the development and application of voting standards;

2. if and to what extent proxy advisors are controlling or significantly influencing shareholder voting without appropriate oversight and without having an actual economic stake in the issuer; and

3. if and to what extent conflicts of interest on the part of proxy advisors could impair informed shareholder voting.

The first point concerns the information quality in the proxy advisory market. Our result suggests that the proxy advisor may send unbiased voting recommendations to shareholders even though it is only a for-profit third party. This offers one justification for the proxy advisor's business model from the information-sale perspective.

Regarding the SEC's second issue, our results suggest that without additional distortions, the influence should not be of concern because the proxy advisor can fulfill its duties as an information intermediary. This is true even if there is no oversight or if the proxy advisor does not have an actual economic stake in the corporation. Nonetheless, our results also caution that this is only an ideal setting and the role of the proxy advisor in facilitating efficient corporate voting can be nuanced.

Regarding the third concern of the SEC, intuitively, the conflicts of interest from the supply side indeed bias the voting recommendations and decrease firm value. However, this is not the only source of the information bias; even an unconflicted proxy advisor can skew its voting recommendations based on the beliefs and preferences of its clients. Moreover, such skewness may even increase firm value.

Finally, we regard this study as a step toward understanding markets for voting recommendations. Many important issues remain open for exploration. For example, our model considers only a monopolistic proxy advisor selling voting recommendations to homogeneous shareholders. It would be of interest to study information bias in the presence of shareholder heterogeneity in various competitive proxy advisory markets (e.g., monopoly vs. duopoly). In addition, shareholder heterogeneity may have interesting implications for competition among proxy advisors. Interesting questions include whether the competition among proxy advisors helps eliminate or reduce the information bias and what its implications are for firm value.

Appendix.

Table A1: Notations

Variables	Description
	Exogenous variables
θ	State, $\theta \in \{0, 1\}$
N	The number of shareholders, N is odd and $N \geq 3$
μ_0	The probability of the state being good: $\Pr(\theta = 1) = \mu_0 \in (0, 1)$
μ	Shareholders' subjective beliefs about the state being good, $\mu \in [0, 1]$
γ	Shareholders lose γ if the proposal passes in a bad state, $\gamma > 0$
p	PA's private signal about the state, $p \in \mathbb{R}$
Φ, φ	PA's Payoff from potential conflicted services, $\varphi = \Phi/N \in \mathbb{R}$
	Endogenous variables
q	Probability of buying recommendations, $q \in [0, 1]$
v	Probability of voting for the proposal, $v \in [0, 1]$
d	Voting outcome, $d \in \{0, 1\}$
u ($\mathcal{U}$)	Shareholders' (expected) utility gross of potential advice subscription fees
Ψ (ψ)	Expected firm value (per share)
ϱ	Threshold of sending positive recommendations
π_1, π_0	Probability of sending state-consistent recommendations
A_1 (A_0)	The gains from following recommendations $r = 1$ ($r = 0$) instead of casting the opposite vote
M	$M = \sqrt[\frac{N-1}{2}]{A_0/A_1}$
G	Defined in equation (13)
r	PA's binary voting recommendation, $r = \mathbb{I}(p \geq \varrho) \in \{0, 1\}$
f	Price of PA's voting recommendation
τ	Probability of uninformed shareholders voting for the proposal, $\tau \in (0, 1)$

Proof of Lemma 1

Before characterizing the voting-stage equilibrium, we make two assumptions to rule out unrealistic equilibria in some off-equilibrium paths.

For one, if all shareholders vote for or against the proposal with probability one, then no shareholder can change the voting outcome unilaterally. Therefore, that all shareholders vote against the proposal even if they all think they should vote for the proposal can still constitute a voting-stage equilibrium. To rule out such cases, we assume that for a given shareholder, if the voting outcome has already been decided by the other shareholders in *all* circumstances, then she votes in favor of the proposal if she thinks a good state is more likely, and vice versa.

Assumption A1. *When a shareholder can never change the voting outcome, the shareholder casts her vote in agreement with her belief.*

Second, to ensure a unique voting-stage equilibrium in the case of indifference, we make the following assumption:

Assumption A2 (Deferential voting)**.** *When indifferent, each shareholder who has purchased the voting recommendation follows the recommendation. When no shareholder has received any voting recommendation ($q = 0$), they all vote in favor of the proposal when they are indifferent.*

Note that this assumption does not specify the voting strategy for indifferent uninformed shareholders in equilibrium. Their voting strategy will be pinned down by the equilibrium conditions.

Further, these two assumptions do not apply on the equilibrium path. On the equilibrium path, as stated in the main text, each shareholder has a positive probability to change the voting result, a positive fraction of shareholders will purchase the voting recommendation and shareholders who have purchased the voting recommendation are not indifferent between voting for and against the proposal. Therefore, these two assumptions are not introduced to select the equilibrium. Instead, we adopt these two assumptions only to present the off-equilibrium path in a concise fashion.

Under Assumptions A1 and A2, we present the complete subgame equilibrium at the voting stage in the following Lemma.

Lemma A1 (Full characterization of voting-stage equilibrium)**.** *For any fixed (q, ϱ), there is a unique voting-stage equilibrium. In particular,*

1. *when $q = 0$, all shareholders vote in favor of the proposal if and only if $\gamma \leq \frac{\mu}{1+\mu}$ for all ϱ;*
2. *when $q \in (0, 1]$, all shareholders vote against the proposal if*

$$\frac{1 - H_1(\varrho)}{1 - H_0(\varrho)} < \frac{\gamma(1 - \mu)}{\mu}$$

and for the proposal if

$$\frac{H_1(\varrho)}{H_0(\varrho)} > \frac{\gamma(1 - \mu)}{\mu};$$

3. *when* $q \in (0, 1]$ *and*
$$\frac{H_1(\varrho)}{H_0(\varrho)} \leq \frac{\gamma(1-\mu)}{\mu} \leq \frac{1-H_1(\varrho)}{1-H_0(\varrho)},$$
a shareholder who has purchased the voting recommendation votes according to the advice of the PA; and

4. *when* $q \in (0, 1)$ *and*
$$\frac{H_1(\varrho)}{H_0(\varrho)} \leq \frac{\gamma(1-\mu)}{\mu} \leq \frac{1-H_1(\varrho)}{1-H_0(\varrho)},$$
an uninformed shareholder votes for the proposal with a probability of τ*, where* $\tau \in (0, 1)$ *is given by equation (8).*

Proof. When $q = 0$, that is, no shareholders purchase the PA's recommendation, Assumptions A1 and A2 take effect. All shareholders cast their votes according to their prior; each shareholder votes for the proposal if and only if $\Pr(\theta = 1) - \gamma \Pr(\theta = 0) = \mu - \gamma(1-\mu) \geq 0$, or equivalently, $\gamma \leq \frac{\mu}{1+\mu}$.

Now, we consider the case in which $q > 0$. We first consider shareholder i who purchases the recommendation and receives $r = 1$. She can change the voting result if and only if she is a pivotal shareholder, that is, the number of "for" votes among other shareholders is exactly $\frac{N-1}{2}$. PIV_i denotes the event of being pivotal. Then, for shareholder i, the payoff of voting for the proposal conditional on being pivotal is as follows:

$$\begin{aligned}
&\Pr(\theta = 1 | r = 1, PIV_i) - \gamma \Pr(\theta = 0 | r = 1, PIV_i) \\
&= \frac{1}{\Pr(r = 1, PIV_i)} \begin{bmatrix} \Pr(PIV_i | \theta = 1, r = 1) \Pr(r = 1 | \theta = 1) \Pr(\theta = 1) \\ -\gamma \Pr(PIV_i | \theta = 0, r = 1) \Pr(r = 1 | \theta = 0) \Pr(\theta = 0) \end{bmatrix} \\
&= \frac{\Pr(PIV_i | r = 1) \left(\Pr(r = 1 | \theta = 1) \Pr(\theta = 1) - \gamma \Pr(r = 1 | \theta = 0) \Pr(\theta = 0) \right)}{\Pr(r = 1, PIV_i)} \\
&= \frac{\Pr(r = 1 | \theta = 1) \Pr(\theta = 1) - \gamma \Pr(r = 1 | \theta = 0) \Pr(\theta = 0)}{\Pr(r = 1)} \\
&= \frac{A_1}{\pi_1 \mu + (1 - \pi_0)(1 - \mu)},
\end{aligned} \tag{A1}$$

where A_1 is given by equation (6). Note that the second line of equation (A1) follows because the true state is unobservable to shareholders, and thus

$$\Pr(PIV_i|\theta = 1, r = 1) = \Pr(PIV_i|\theta = 0, r = 1) = \Pr(PIV_i|r = 1).$$

For shareholder i to vote for the proposal, equation (A1) must be nonnegative, which holds when $A_1 \geq 0$, equivalent to

$$\frac{1 - H_1(\varrho)}{1 - H_0(\varrho)} \geq \frac{\gamma(1 - \mu)}{\mu}. \tag{A2}$$

Note that when $A_1 = 0$, by Assumption A2, shareholder i votes for the proposal. Further we know that if $\frac{1-H_1(\varrho)}{1-H_0(\varrho)} < \frac{\gamma(1-\mu)}{\mu}$, the shareholder who acquires a positive signal is not willing to vote for the proposal. Because the shareholder with a positive recommendation is the most likely type to vote in favor of the proposal, no shareholders are willing to vote for the proposal when $\frac{1-H_1(\varrho)}{1-H_0(\varrho)} < \frac{\gamma(1-\mu)}{\mu}$.

Similarly, if shareholder i subscribes to the voting recommendation and receives $r = 0$, the payoff of voting for the proposal conditional on being pivotal is as follows:

$$\Pr(\theta = 1|r = 0, PIV_i) - \gamma \Pr(\theta = 0|r = 0, PIV_i) = \frac{-A_0}{(1 - \pi_1)\mu + \pi_0(1 - \mu)}, \tag{A3}$$

where A_0 is given by equation (7). For the shareholder to follow the recommendation and vote against the proposal, equation (A3) needs to be nonpositive. This holds when $A_0 \geq 0$, equivalent to

$$\frac{H_1(\varrho)}{H_0(\varrho)} \leq \frac{\gamma(1 - \mu)}{\mu}. \tag{A4}$$

Note that when $A_0 = 0$, by Assumption A2, shareholder i votes against the proposal. By the same logic as before, when $\frac{H_1(\varrho)}{H_0(\varrho)} > \frac{\gamma(1-\mu)}{\mu}$, all shareholders will vote against the proposal.

Note that equations (A2) and (A4) can weakly hold at the voting stage if we consider off-equilibrium path of play. However, in the main text, we prove by contradiction that equations (A2) and (A4) must strictly hold in the equilibrium of the whole game.

Next, when $q \in (0, 1)$, for the uninformed shareholder, the payoff of voting for the proposal, conditional on the event of being pivotal, is as follows:

$$
\begin{aligned}
&\Pr(\theta=1|PIV_i)-\gamma\Pr(\theta=0|PIV_i)\\
&=\frac{1}{\Pr(PIV_i)}\Big(\Pr(PIV_i|\theta=1,r=1)\Pr(\theta=1,r=1)+\Pr(PIV_i|\theta=1,r=0)\Pr(\theta=1,r=0)\\
&\quad-\gamma\left(\Pr(PIV_i|\theta=0,r=1)\Pr(\theta=0,r=1)+\Pr(PIV_i|\theta=0,r=0)\Pr(\theta=0,r=0)\right)\Big)\\
&=\frac{1}{\Pr(PIV_i)}\Big(\Pr(PIV_i|r=1)A_1-\Pr(PIV_i|r=0)A_0\Big). \qquad \text{(A5)}
\end{aligned}
$$

Again, in the main text we prove by contradiction that in equilibrium uninformed shareholders must use a (strictly) mixed voting strategy. So, by setting equation (A5) equal to zero generates the information-purchase strategy of the uninformed shareholder.

Let $\tau\in(0,1)$ be the uninformed shareholders' probability of voting for the proposal. Given that the fraction of shareholders who purchase the PA's recommendation is q, for a generic shareholder, the probability of voting for the proposal conditioning on the event $r=1$ is $\Pr(v_i=1|r=1)=q+(1-q)\tau$. So, when $r=1$, the probability of a shareholder being pivotal is

$$
\Pr(PIV_i|r=1)=\binom{N-1}{\frac{N-1}{2}}(q+(1-q)\tau)^{\frac{N-1}{2}}((1-q)(1-\tau))^{\frac{N-1}{2}}, \qquad \text{(A6)}
$$

where

$$
\binom{N-1}{\frac{N-1}{2}}=\frac{(N-1)!}{\left(\frac{N-1}{2}\right)!\left(\frac{N-1}{2}\right)!}
$$

is the binomial coefficient. Similarly, for a generic shareholder, the probability of voting for the proposal conditional on the event $r=0$ is $\Pr(v_i=1|r=0)=(1-q)\tau$, and thus when $r=0$ the probability of a shareholder being pivotal is

$$
\Pr(PIV_i|r=0)=\binom{N-1}{\frac{N-1}{2}}((1-q)\tau)^{\frac{N-1}{2}}(1-(1-q)\tau)^{\frac{N-1}{2}}. \qquad \text{(A7)}
$$

Using M as defined by equation (9), plugging equations (A6) and (A7) into equation (A5), and setting it to be zero yields the following:

$$\frac{(q+(1-q)\tau)\,(1-q)(1-\tau)}{(1-q)\tau\,(1-(1-q)\tau)} = M. \tag{A8}$$

Solving equation (A8) we obtain τ as given by equation (8) (the root that does not lie between 0 and 1 is omitted). Notice that τ is continuous at $\gamma = \frac{\mu}{1-\mu}$ because

$$\lim_{\gamma \to \frac{\mu}{1-\mu}} \frac{(1-M-2q)+\sqrt{(1-M)^2+4q^2M}}{2(1-q)(1-M)} = \frac{1}{2}.$$

We further confirm that $\tau \in (0,1)$. First, we show that $\tau > 0$. For the numerator of τ, given that $\left(\sqrt{(1-M)^2+4q^2M}\right)^2 - (1-M-2q)^2 = 4q(1-q)(1-M)$, the numerator of τ always has the same sign as its denominator does. So we have $\tau > 0$. Second, to show $\tau < 1$, we study that

$$1-\tau = \frac{\sqrt{(1-M)^2+4q^2M}-(1-M+2qM)}{2(1-q)(M-1)}.$$

Because $\left(\sqrt{(1-M)^2+4q^2M}\right)^2 - (1-M+2qM)^2 = 4Mq(1-q)(M-1)$, the numerator of $1-\tau$ always has the same sign as its denominator does. Therefore $\tau < 1$.

In summary, we have shown that when $q \in (0,1]$ and inequalities (A2) and (A4) both hold, shareholders who have purchased the voting recommendation follow it and uninformed shareholders vote for the proposal with probability $\tau \in (0,1)$ (note when $q = 1$, there are no uninformed shareholders). In addition, when $q \in (0,1]$ and inequality (A2) is violated, all shareholders vote against the proposal. When $q \in (0,1]$ and inequality (A4) is violated, all shareholders vote for the proposal. $\square$

While Lemma A1 presents the full characterization of the voting-stage equilibrium, only the case in which $q \in (0,1)$ and both $A_1 > 0$ and $A_0 > 0$ (inequalities (A2) and (A4) are strict) can be on the equilibrium path; otherwise if $q = 1$ or $A_1 = 0$ or $A_0 = 0$, as argued in the main text the voting recommendation cannot change the voting behavior and is of no value to shareholders. Therefore, a profit-maximizing PA will not choose such subgames. Note that the PA will not allow $q = 0$ either. As shown in the Proof of Proposition 1, $\lim_{q\to 0} \frac{\partial \Pi}{\partial q} > 0$ if $\gamma \neq \frac{\mu}{1-\mu}$, where Π is the PA's total profits, and $q^* = \frac{1}{\sqrt{N}} > 0$ if $\gamma = \frac{\mu}{1-\mu}$. So, $q = 0$ is always a dominated strategy for the PA and will be off the equilibrium. $\square$

Before moving on, we summarize some properties of M and τ on the equilibrium path in the following lemma for future use.

Lemma A2. *For all* $0 < q < 1$ *and* $M > 0$, $M < 1$ *when* $\gamma < \frac{\mu}{1-\mu}$, *and* $M > 1$ *when* $\gamma > \frac{\mu}{1-\mu}$. *Further we have the following.*

1. *Assume* $\gamma > \frac{\mu}{1-\mu}$, *we have* $\frac{\partial \tau}{\partial q} > 0$. *When* $\varrho < \hat{\varrho}$, *where* $\hat{\varrho}$ *is given by equation* (15), $\frac{\partial M}{\partial \varrho} < 0$ *and* $\frac{\partial \tau}{\partial \varrho} > 0$, *whereas when* $\varrho > \hat{\varrho}$, $\frac{\partial M}{\partial \varrho} > 0$ *and* $\frac{\partial \tau}{\partial \varrho} < 0$.
2. *Assume* $\gamma < \frac{\mu}{1-\mu}$, *we have* $\frac{\partial \tau}{\partial q} < 0$. *When* $\varrho < \hat{\varrho}$, $\frac{\partial M}{\partial \varrho} > 0$ *and* $\frac{\partial \tau}{\partial \varrho} < 0$, *whereas when* $\varrho > \hat{\varrho}$, $\frac{\partial M}{\partial \varrho} < 0$ *and* $\frac{\partial \tau}{\partial \varrho} > 0$.

Proof. First, given that $A_0 - A_1 = \gamma(1-\mu) - \mu$, we know that $M < 1$ when $\gamma < \frac{\mu}{1-\mu}$, and $M > 1$ when $\gamma > \frac{\mu}{1-\mu}$. Next, taking derivative of equation (8) with respect to q yields the following:

$$\frac{\partial \tau}{\partial q} = -\frac{(M+1)\sqrt{(1-M)^2 + 4q^2M} - (1-M)^2 - 4qM}{2(1-M)(1-q)^2\sqrt{(1-M)^2 + 4q^2M}} = \begin{cases} < 0 & \text{if } \gamma < \frac{\mu}{1-\mu}, \\ > 0 & \text{if } \gamma > \frac{\mu}{1-\mu}. \end{cases} \tag{A9}$$

$\frac{\partial \tau}{\partial q}$ has the same sign as $M-1$ because $(M+1)^2[(1-M)^2 + 4q^2M] - [(1-M)^2 + 4qM]^2$ $= 4M(1-M)^2(1-q)^2 > 0$.

Note ϱ affects τ only through M. Taking derivative of equation (8) with respect to M yields the following:

$$\frac{\partial \tau}{\partial M} = -\frac{q\left(\sqrt{(1-M)^2 + 4q^2M} - (M+1)q\right)}{(1-M)^2(1-q)\sqrt{(1-M)^2 + 4q^2M}} < 0, \tag{A10}$$

as $(1-M)^2 + 4q^2M - (M+1)^2q^2 = (1-M)^2(1-q^2) > 0$. Based on equation (9),

$$\frac{\partial M}{\partial \varrho} = \frac{2M}{N-1}\frac{(\mu - \gamma(1-\mu))(\gamma(1-\mu)h_0(\varrho) - \mu h_1(\varrho))}{A_0A_1}. \tag{A11}$$

The signs of $\frac{\partial \tau}{\partial q}$, $\frac{\partial \tau}{\partial \varrho}$, and $\frac{\partial M}{\partial \varrho}$ are thus easy to check. $\square$

Proof of Lemma 2

At the information-purchase stage, a shareholder will use a strictly mixed strategy on information acquisition, that is, $q \in (0,1)$, only if the value of recommendation is equal to its price. To determine the information-purchase decision, we first calculate the value of the recommendation. Suppose shareholder i purchases the PA's recommendation. If she receives $r = 1$, shareholder i's payoff of voting "for" for sure versus voting "for" with probability τ is as follows:

$$\begin{aligned} V(q, \varrho|r=1) &= (1-\tau)\left[\Pr(\theta=1|PIV_i, r=1) - \gamma \Pr(\theta=0|PIV_i, r=1)\right]\Pr(PIV_i|r=1) \\ &= \frac{(1-\tau)\Pr(PIV_i|r=1)}{\Pr(r=1)} A_1, \end{aligned} \tag{A12}$$

Note that because the payoff of voting conditional on the nonpivotal case is zero, there is only one term in equation (A12). Similarly, if she receives $r = 0$, shareholder i's payoff of voting "against" for sure versus voting "against" with probability $1-\tau$ is as follows:

$$V(q, \varrho|r=0) = \frac{\tau \Pr(PIV_i|r=0)}{\Pr(r=0)} A_0, \tag{A13}$$

Therefore, the overall value of the recommendation is the weighted average of equations (A12) and (A13), as follows:

$$\begin{aligned} V(q, \varrho) &= \Pr(r=1)V(q, \varrho|r=1) + \Pr(r=0)V(q, \varrho|r=0) \\ &= (1-\tau)\Pr(PIV_i|r=1)A_1 + \tau \Pr(PIV_i|r=0)A_0 \\ &= A_1 \Pr(PIV_i|r=1) \\ &= A_1 \times \binom{N-1}{\frac{N-1}{2}} [q + (1-q)\tau]^{\frac{N-1}{2}} [(1-q)(1-\tau)]^{\frac{N-1}{2}}. \end{aligned} \tag{A14}$$

Note that the third equality uses the fact that equation (A5) is zero in equilibrium, and the fourth equality uses equations (6) and (A6).

When deciding whether to purchase the voting advice, shareholder i compares the value of the voting recommendation, $V(q, \varrho)$, with its price f. She purchases the advice if and only if $V(q,\varrho) > f$ and does not do so if and only if $V(q,\varrho) < f$. When $V(q,\varrho) = f$, the shareholder uses a mixed purchasing strategy. We thus solve the interior q^* by setting $V(q,\varrho) = f$. Using G as defined by equation (13), $V(q,\varrho) = f$ can be rewritten as the following:

$$(q + (1-q)\tau)(1-q)(1-\tau) = G. \tag{A15}$$

To solve for information demand q given a recommendation price f, we will discuss the cases in which $\gamma \neq \frac{\mu}{1-\mu}$ and $\gamma = \frac{\mu}{1-\mu}$ next.

Case 1: $\gamma \neq \frac{\mu}{1-\mu}$**.** When $\gamma \neq \frac{\mu}{1-\mu}$ (and thus $M \neq 1$), plugging equation (8) into equation (A15), we have the following:

$$\frac{Mq\left(\sqrt{(1-M)^2 + 4Mq^2} - q - Mq\right)}{(1-M)^2} = G. \tag{A16}$$

Denote the left-hand side of equation (A16) $F(q)$. We first show that $F(q)$ is single peaked for $q \in (0,1)$. Because $(1-M)^2 + 4Mq^2 - (q+Mq)^2 = (1-M)^2(1-q^2) \geq 0$, $F(q) \geq 0$. Confirming that $F(0) = 0$ and $F(1) = 0$ is easy to do. Taking the derivative of $F(q)$ with respect to q and taking their limits, we have

$$\lim_{q\to 0} \frac{dF}{dq} = \frac{M}{\sqrt{(1-M)^2}} > 0$$

and

$$\lim_{q\to 1} \frac{dF}{dq} = -\frac{M}{1+M} < 0.$$

Taking the second derivative of $F(q)$ with respect to q, we obtain the following:

$$\begin{aligned}
\frac{\mathrm{d}^2 F}{\mathrm{d}q^2} &= \frac{2M}{(1-M)^2}\left(-1 - M + \frac{16M^2q^3 + 6qM(1-M)^2}{((1-M)^2 + 4Mq^2)^{\frac{3}{2}}}\right) \\
&< \frac{2M}{(1-M)^2}\left(-1 - M + \frac{16M^2 + 6M(1-M)^2}{((1-M)^2 + 4M)^{\frac{3}{2}}}\right) \\
&= -\frac{2M(1+M^2)}{(1+M)^3} < 0.
\end{aligned}$$

The inequality follows because

$$\frac{\partial}{\partial q}\left(\frac{16M^2q^3+6qM(1-M)^2}{((1-M)^2+4Mq^2)^{\frac{3}{2}}}\right)=\frac{6M(1-M)^4}{((1-M)^2+4Mq^2)^{\frac{5}{2}}}>0.$$

All considered, $F(q)$ increases from 0 and then decreases to 0 as q moves from 0 to 1, and it reaches its maximum at an interior point. The maximum point $\tilde{q}$ (the improper roots are neglected) is as follows:

$$\tilde{q}=\begin{cases}\frac{\sqrt{1-M}}{2} & \text{if } \gamma<\frac{\mu}{1-\mu},\\ \frac{1}{2}\sqrt{\frac{M-1}{M}} & \text{if } \gamma>\frac{\mu}{1-\mu}.\end{cases}\tag{A17}$$

The corresponding price of the voting recommendation $\tilde{f}$ can be computed as follows:

$$\tilde{f}=\begin{cases}\binom{N-1}{\frac{N-1}{2}}A_1\left(\frac{M}{4}\right)^{\frac{N-1}{2}} & \text{if } \gamma<\frac{\mu}{1-\mu},\\ \binom{N-1}{\frac{N-1}{2}}A_1\left(\frac{1}{4}\right)^{\frac{N-1}{2}} & \text{if } \gamma>\frac{\mu}{1-\mu},\end{cases}\tag{A18}$$

where A_1 is given by equation (6). Therefore, when $f>\tilde{f}$, the price of the PA's recommendation is too prohibitive and thus $q^*=0$. When $f=\tilde{f}$, there is a unique solution to equation (A16), which is denoted by $\tilde{q}$ as in equation (A17). When $f<\tilde{f}$, there are two solutions to equation (A16) and they are given as follows:

$$\begin{cases}q^{\mathrm{I}}=\sqrt{\frac{M-2G(M+1)+\sqrt{M(1-4G)(M-4G)}}{2M}},\\ q^{\mathrm{II}}=\sqrt{\frac{M-2G(M+1)-\sqrt{M(1-4G)(M-4G)}}{2M}}.\end{cases}\tag{A19}$$

The reason we may have two equilibria at this stage is as follows. When contemplating whether to subscribe to the PA's voting advice, each shareholder faces the following trade-off. On the one hand, the purchased voting advice helps the shareholder to vote in a more informed way and, hence, improve her payoff. On the other hand, every shareholder has an incentive to free ride given the collective nature of voting. When the probability of purchasing information q is small, the chance of free riding is slim and the marginal benefit of purchasing the voting recommendation is large. The converse is true when q becomes large. Thus, the value of the PA's voting recommendation $V(q, \varrho)$ is increasing in q when q is small and is decreasing in q when q is large. Therefore, when the price of the voting recommendation is moderate, that is, $f < \tilde{f}$, we can find two q's that satisfy $V(q, \varrho) = f$ in general.

To have a well-defined problem of information pricing, we need to decide which equilibrium is selected in the event of multiple equilibria at the information-purchase stage. The two equilibria, q^{I} and q^{II}, can be ranked with respect to shareholder welfare, that is, shareholders' expected utility (net of the potential fee paid for the recommendations). Following Malenko and Malenko (2019), we make the following assumption to select the equilibrium:

Assumption A3 (Equilibrium Selection)**.** *When $f < \tilde{f}$, shareholders coordinate on the equilibrium that maximizes the total shareholder welfare.*

Because information prices are the same in the two equilibria given by equation (A19), according to Assumption A3, the equilibrium with the higher expected utility (gross of the potential fee paid for the recommendations) should be selected. As shown in Lemma A4, shareholders' expected utility (gross of the potential fee paid for the recommendations) $\mathcal{U}$ is monotonically increasing in q. Hence, the equilibrium with a higher q is always associated with a higher $\mathcal{U}$ for shareholders. Or simply, q^{I} is selected. We further check that when $f = \tilde{f}$, q^{I} is equivalent to equation (A17). Finally, we write q^{I} as $q(f, \varrho)$ in equation (12).

Case 2: $\gamma = \frac{\mu}{1-\mu}$**.** When $\gamma = \frac{\mu}{1-\mu}$ (thus $M = 1$ and $\tau = \frac{1}{2}$), equation (A15) can be simplified to $\frac{1}{4}(1 - q^2) = G$, which yields two roots $q = \sqrt{1 - 4G}$ and $q = 0$. We select the root with larger q, so here we select $q = \sqrt{1 - 4G}$. Note that equation (A19) also applies to the case because based on it, $q^{\mathrm{I}} = \sqrt{1 - 4G}$ and $q^{\mathrm{II}} = 0$, and we select q^{I} as the equilibrium information demand. So, $q(f, \varrho)$ in equation (12) can still summarize the information demand. $\square$

Proof of Proposition 1

For ease of exposition, denote Π as the PA's per capita profit as in equation (5). As analyzed in the main text, the PA's problem becomes choosing the information demand q and the cutoff point ϱ to maximize its profits: $\max_{q,\varrho} \Pi = qV + [\pi_1\mu_0 + (1 - \pi_0)(1 - \mu_0)]\varphi$.

To compute the optimal q, we take the derivative of the profit function with respect to q and set it to zero, as follows:

$$0 = [q + (1-q)\tau](1-q)(1-\tau) + q\frac{N-1}{2}\left(1 - \tau + (1-q)\frac{\partial \tau}{\partial q}\right)((1-q)(1-2\tau) - q)\,.$$

Inserting equations (8) and (A9) into the above equation, we obtain that the RHS of the above equation is positively related to $g_1(q)$, where

$$g_1(q) \equiv \frac{qM\Gamma(q)}{2(1-M)^2\sqrt{(1-M)^2 + 4q^2M}}, \tag{A20}$$

and $\Gamma(q) = (1+N)(1-M)^2 + 8MNq^2 - 2Nq(M+1)\sqrt{(1-M)^2 + 4q^2M}$.

Next, we show that $g_1(q)$ is single peaked for $q \in (0,1)$. Note that given $q \in (0,1)$, the sign of $g_1(q)$ is the same as the sign of $\Gamma(q)$. To determine the sign of $\Gamma(q)$, we first know that $\lim_{q\to 0}\Gamma(q) = (1-M)^2(N+1) > 0$ and $\lim_{q\to 1}\Gamma(q) = -(1-M)^2(N-1) < 0$. By the intermediate value theorem, there exists at least one solution to $g_1(q) = 0$ when $q \in (0,1)$. Taking the derivative of $\Gamma(q)$ with respect to q yields that $\Gamma'(q)$ is positively related to the following negative term:

$$8qM\sqrt{(1-M)^2 + 4q^2M} - \left[1 + M^3 + M(M+1)(8q^2 - 1)\right] < 0.$$

The inequality follows from the fact that for any $q \in (0,1)$ and $M > 0$,

$$\begin{aligned}\left(8qM\sqrt{(1-M)^2 + 4q^2M}\right)^2 &- \left[1 + M^3 + M(M+1)(8q^2 - 1)\right]^2 \\ &= -64M^2(1-M)^2q^4 - 16M(1-M)^4q^2 - (1-M)^4(1+M)^2 < 0.\end{aligned}$$

Therefore, given any ϱ the solution to $g_1(q) = 0$ is unique and it maximizes the PA's profit function Π. Solving $g_1(q) = 0$ yields (the improper roots are neglected) equation (16), that is, the optimal q^* depends on the optimal choice of ϱ. Thus, the PA's problem becomes to choose ϱ to maximize profits subject to the constraint equation (16).

Next, we solve for the optimal cutoff point ϱ^*, depending on whether or not the PA is conflicted. Before the derivation, we prepare an auxiliary lemma as follows.

Lemma A3. *When $\gamma \leq (\geq) \frac{\mu}{(1-\mu)}$, we have $M \leq (\geq) 1$, $\sqrt{(1-M)^2 + 4Mq^2} \geq (\leq) 2Mq$, and $2q \geq (\leq) \sqrt{(1-M)^2 + 4Mq^2}$.*

Proof. When $\gamma \leq \frac{\mu}{(1-\mu)}$, based on equation (9) we know that $M \leq 1$, thus $K \equiv (1-M)^2 + 4Mq^2 - (2Mq)^2 = (1-M)^2 + 4q^2 M(1-M) \geq 0$. So $\sqrt{(1-M)^2 + 4Mq^2} \geq 2Mq$. To show $2q \geq \sqrt{(1-M)^2 + 4Mq^2}$, it is equivalent to show $4q^2 \geq 1 - M$. Plugging equation (16) into $4q^2 \geq 1 - M$, we obtain the following:

$$\begin{aligned}
4q^2 \geq 1 - M &\Leftrightarrow (1+M)\sqrt{4M + N^2(1-M)^2} \geq (-M^2N - 4M + N) \\
&\Leftrightarrow \left((1+M)\sqrt{4M + N^2(1-M)^2}\right)^2 \geq (-M^2N - 4M + N)^2 \\
&\Leftrightarrow (-8N+4)M^3 - 8M^2 + (8N+4)M \geq 0.
\end{aligned}$$

One can verify that the last expression above holds for all $M < 1$ and $N \geq 3$.

When $\gamma \geq \frac{\mu}{(1-\mu)}$, based on equation (9) we know that $M \geq 1$. We obtain the following by plugging equation (16) into K:

$$K = \frac{(M-1)\left(4M + N - M^2N + (1+M)\sqrt{4M + N^2 - 2MN^2 + M^2N^2}\right)}{2N},$$

which has the same sign as the term $M - 1$ because

$$\begin{aligned}
&(4M + N - M^2N)^2 - (1+M)^2(4M + N^2 - 2MN^2 + M^2N^2) \\
&\qquad = 4(M-1)M(M - 1 + 2N(M+1)) \geq 0
\end{aligned}$$

when $M \geq 1$. Thus, we have $\sqrt{(1-M)^2 + 4Mq^2} \leq 2Mq$ when $\gamma \geq \frac{\mu}{(1-\mu)}$. Finally, $2q \leq \sqrt{(1-M)^2 + 4Mq^2}$ holds because $2q \leq 2q\sqrt{M} \leq \sqrt{(1-M)^2 + 4Mq^2}$ if $M \geq 1$. □

Case 1: $\varphi = 0$**.** Based on equations (13) and (A14), we have

$$\Pi = q \times A_1 G^{\frac{N-1}{2}} \binom{N-1}{\frac{N-1}{2}}.$$

To derive the optimal ϱ^*, we utilize the fact that the monotonicity of the PA's profit function Π is the same as that of $\log(\Pi)$. We take the partial derivative of $\log(\Pi)$ with respect to ϱ and obtain the following:

$$\begin{aligned}\frac{\partial \log(\Pi)}{\partial \varrho} &= \frac{1}{A_1}\frac{\partial A_1}{\partial \varrho} + \frac{N-1}{2}\frac{1}{G}\frac{\partial G}{\partial \varrho}, \\ &= \frac{1}{A_1}\frac{\partial A_1}{\partial \varrho} + \frac{N-1}{2}\frac{1}{G}\frac{\partial G}{\partial M}\frac{\partial M}{\partial \varrho},\end{aligned} \tag{A21}$$

where A_1 and G are given by equations (6) and (13), respectively. Further, in the first term of equation (A21),

$$\frac{\partial A_1}{\partial \varrho} = \gamma(1-\mu)h_0(\varrho) - \mu h_1(\varrho).$$

It is positive (negative) when $\varrho < (>)\hat{\varrho}$, where $\varrho = \hat{\varrho}$ is the solution to equation (15). In the second term of equation (A21),

$$\frac{\partial G}{\partial M} = \frac{q[2M^2q^2 + 6Mq^2 + (1-M)^2 - q(3M+1)\sqrt{(1-M)^2 + 4Mq^2}]}{(1-M)^3\sqrt{(1-M)^2+4Mq^2}}. \tag{A22}$$

To verify the sign of $\frac{\partial G}{\partial M}$, notice that

$$\begin{aligned}&\left(2M^2q^2 + 6Mq^2 + (1-M)^2\right)^2 - \left(q(3M+1)\sqrt{(1-M)^2+4Mq^2}\right)^2 = \\ &\qquad (1-q)(1+q)(1-M)^2[(1-M)^2 + 4Mq^2 - 4M^2q^2].\end{aligned}$$

By Lemma A3, the sign of $(1-M)^2 + 4Mq^2 - 4M^2q^2$ is the same as $1-M$. Therefore, the numerator and the denominator of equation (A22) share the same sign. Hence, $\frac{\partial G}{\partial M} > 0$ for all $M \neq 1$.

Now we are ready to discuss the property of equation (A21) in the following cases. When $\gamma < \frac{\mu}{1-\mu}$, $\frac{\partial M}{\partial \varrho}$ is positive (negative) if $\varrho < (>)\hat{\varrho}$ (see Lemma A2). Therefore, Π is single-peaked around $\hat{\varrho}$ with respect to ϱ when $\gamma < \frac{\mu}{1-\mu}$.

When $\gamma > \frac{\mu}{1-\mu}$, we plug equation (A11) into equation (A21) and obtain the following:

$$\frac{\partial \log(\Pi)}{\partial \varrho} = \frac{1}{A_1}\frac{\partial A_1}{\partial \varrho} \times \left(1 + \frac{M}{G}\frac{\partial G}{\partial M}\left(\frac{\frac{\partial A_0}{\partial \varrho}A_1}{\frac{\partial A_1}{\partial \varrho}A_0} - 1\right)\right).$$

Note that $\frac{\partial A_1}{\partial \varrho} > (<) 0$ when $\varrho < (>) \hat{\varrho}$. Therefore, to show Π is single-peaked around $\hat{\varrho}$ with respect to ϱ, we only need to show that

$$\frac{M}{G}\frac{\partial G}{\partial M}\left(1 - \frac{\frac{\partial A_0}{\partial \varrho}A_1}{\frac{\partial A_1}{\partial \varrho}A_0}\right) < 1,$$

which can be simplified to

$$\frac{M}{G}\frac{\partial G}{\partial M}\left(1 - \frac{1}{M^{\frac{N-1}{2}}}\right) < 1 \tag{A23}$$

by using that $M^{\frac{N-1}{2}} = \frac{A_0(\partial A_1/\partial \varrho)}{A_1(\partial A_0/\partial \varrho)}$.

Because $M > 1$ when $\gamma > \frac{\mu}{1-\mu}$, $\frac{G}{M} > \frac{\partial G}{\partial M}$ is a sufficient condition to equation (A23). Inserting equations (A16) and (A22) into $\frac{G}{M} > \frac{\partial G}{\partial M}$, we have

$$\begin{aligned}
\frac{G}{M} > \frac{\partial G}{\partial M} &\Leftrightarrow \left[\begin{array}{c} \frac{q\left(\sqrt{(1-M)^2+4Mq^2}-q-Mq\right)}{(1-M)^2} > \\ \frac{q[2M^2q^2+6Mq^2+(1-M)^2-q(3M+1)\sqrt{(1-M)^2+4Mq^2}]}{(1-M)^3\sqrt{(1-M)^2+4Mq^2}} \end{array}\right] \\
&\Leftrightarrow \left[\begin{array}{c} \sqrt{(1-M)^2+4Mq^2} - q - Mq > \\ \frac{q(3M+1)\sqrt{(1-M)^2+4Mq^2}-\left(2M^2q^2+6Mq^2+(1-M)^2\right)}{(M-1)\sqrt{(1-M)^2+4Mq^2}} \end{array}\right] \\
&\Leftrightarrow \left[\begin{array}{c} M((1-M)^2+4Mq^2) - q(M^2-1)\sqrt{(1-M)^2+4Mq^2} > \\ q(3M+1)\sqrt{(1-M)^2+4Mq^2} - \left(2M^2q^2+2Mq^2\right) \end{array}\right] \\
&\Leftrightarrow \left[\begin{array}{c} (1-M)^2+4Mq^2+\left(2Mq^2+2q^2\right) > \\ q(M+3)\sqrt{(1-M)^2+4Mq^2} \end{array}\right] \\
&\Leftrightarrow (1-q)(1+q)\,(M-1)^3\,(4q^2+M-1) > 0.
\end{aligned}$$

The last line follows $M > 1$ and $q \in (0,1)$. Therefore, Π is single-peaked around $\hat{\varrho}$ with respect to ϱ when $\gamma > \frac{\mu}{1-\mu}$.

Finally, when $\gamma = \frac{\mu}{1-\mu}$, we have $M = 1$, $\tau = \frac{1}{2}$, and equation (A21) is reduced to

$$\frac{\partial \log(\Pi)}{\partial \varrho} = \frac{1}{A_1}\left(\mu h_0(\varrho) - \gamma(1-\mu)h_1(\varrho)\right).$$

Therefore, when $\frac{h_1(\varrho)}{h_0(\varrho)} < \frac{\gamma(1-\mu)}{\mu} = 1$, $\frac{\partial \log(\Pi)}{\partial \varrho} > 0$ and when $\frac{h_1(\varrho)}{h_0(\varrho)} > \frac{\gamma(1-\mu)}{\mu} = 1$, $\frac{\partial \log(\Pi)}{\partial \varrho} > 0$. So Π is also single-peaked around $\hat{\varrho}$ with respect to ϱ when $\gamma = \frac{\mu}{1-\mu}$.

In sum, when $\varphi = 0$, the optimal value for ϱ is $\varrho^* = \hat{\varrho}$, where $\hat{\varrho}$ is the solution to equation (15). Further, $q^* = q^*(\hat{\varrho})$, where $q^*(\cdot)$ is given by equation (12).

Case 2: $\varphi \neq 0$**.** When $\varphi \neq 0$, the PA's problem can be rewritten as

$$\max_{q,\varrho} \Pi = qV + [\pi_1\mu_0 + (1-\pi_0)(1-\mu_0)]\varphi.$$

Taking the derivative of Π with respect to q and setting it to zero yields that the first-order condition is positively related to $g_1(q)$ as in equation (A20). Thus, as in the first case, we have $q^* = q^*(\varrho^*)$, where $q^*(\cdot)$ is given by equation (16).

To solve for ϱ^* when $\varphi \neq 0$, we take derivative of Π with respect to ϱ and obtain the following:

$$\frac{\partial \Pi}{\partial \varrho} = \frac{\partial (qV)}{\partial \varrho} + \frac{\partial}{\partial \varrho}\left(\pi_1\mu_0 + (1-\pi_0)(1-\mu_0)\right)\varphi. \tag{A24}$$

From the case in which $\varphi = 0$, $\frac{\partial(qV)}{\partial \varrho} = 0$ if $\varrho = \hat{\varrho}$. Plugging $\varrho = \hat{\varrho}$ into equation (A24) yields

$$\frac{\partial \Pi}{\partial \varrho} = \frac{\partial}{\partial \varrho}\left(\pi_1\mu_0 + (1-\pi_0)(1-\mu_0)\right)\varphi = \left(-\mu_0 h_1(\hat{\varrho}) - (1-\mu_0)h_0(\hat{\varrho})\right)\varphi.$$

Therefore, when $\varphi > (<)0$, at $\varrho = \hat{\varrho}$, $\frac{\partial \Pi}{\partial \varrho} < (>)0$, and we have $\varrho^* < (>)\hat{\varrho}$; that is, the PA is more (less) likely to issue positive recommendations when there are conflicts of interest.

Further, based on equation (16), we have

$$\begin{aligned}\frac{\partial q}{\partial \varrho} = \frac{\partial q}{\partial M}\frac{\partial M}{\partial \varrho} = \frac{\partial M}{\partial \varrho} &\times \frac{(M-1)N}{4\sqrt{2}(MN)^{\frac{3}{2}}\sqrt{4M + N^2(M-1)^2}} \\ &\times \frac{2M + N^2 + M^2N^2 - N(M+1)\sqrt{4M + N^2(M-1)^2}}{\sqrt{4M - N(M-1)^2 + (M+1)\sqrt{4M + N^2(M-1)^2}}}.\end{aligned} \tag{A25}$$

Because $(2M + N^2 + M^2N^2)^2 - N^2(M+1)^2(4M + N^2(M-1)^2) = 4M^2(N^2-1)^2 > 0$, the derivative $\frac{\partial q}{\partial M}$ has the same sign as $M - 1$. So based on the property of M, we know that $\frac{\partial q}{\partial M} > (<) 0$ if $\gamma > (<) \frac{\mu}{1-\mu}$. Based on equation (A11), we know that for $\varrho < \hat{\varrho}$, when $\gamma > (<) \frac{\mu}{1-\mu}$, $\frac{\partial M}{\partial \varrho} < (>) 0$, whereas for $\varrho > \hat{\varrho}$, when $\gamma > (<) \frac{\mu}{1-\mu}$, $\frac{\partial M}{\partial \varrho} > (<) 0$. Considered together, we have $\frac{\partial q}{\partial \varrho} < 0$ if $\varrho < \hat{\varrho}$ whereas $\frac{\partial q}{\partial \varrho} > 0$ if $\varrho > \hat{\varrho}$. Further note that when $\varphi > 0$, $\varrho^* < \hat{\varrho}$, so $q^* > q(\hat{\varrho})$. And when $\varphi < 0$, $\varrho^* > \hat{\varrho}$, so $q^* > q(\hat{\varrho})$ as well.

Finally, when $\mu_0 = \frac{1}{2}$, we have $\gamma = \frac{\mu}{1-\mu}$ and $M = 1$. Based on equation (16), we know $q^* = \frac{1}{\sqrt{N}}$. In other words, when the true prior is neutral between the states, the fraction of informed shareholders only depends on the total number of shares N, not related to φ, γ, or μ. To sum, $q^* \geq q^*(\hat{\varrho})$. □

Proof of Unbiased Recommendations

With p's distribution, we have the following posterior probabilities:

$$\Pr(\theta = 1 | p, \mu_0) = \frac{\mu_0 h_1(p)}{\mu_0 h_1(p) + (1-\mu_0) h_0(p)},$$
$$\Pr(\theta = 0 | p, \mu_0) = \frac{(1-\mu_0) h_1(p)}{\mu_0 h_1(p) + (1-\mu_0) h_0(p)}.$$

Thus, $\Pr(\theta = 1|p, \mu_0) \geq \Pr(\theta = 0|p, \mu_0)$ if and only if $\frac{h_1(p)}{h_0(p)} \geq \frac{1-\mu_0}{\mu_0}$. According to Definition 2, the cutoff point of the unbiased recommendation is ϱ^{B} as defined in equation (17). □

Proof of Lemma 3

Assuming whether or not the proposal is implemented is directly determined by the recommendation. Suppose that the recommendation is characterized by ϱ as follows: $r(p) = \mathbb{I}\{p \geq \varrho\}$. Based on equation (18), for any ϱ, firm value is computed as follows: $\Psi = \mu_0 (1 - H_1(\varrho)) - (1-\mu_0)(1 - H_0(\varrho))$. Taking the derivative of firm value with respect to ϱ yields the optimal ϱ^0, which is the solution to equation (17). Therefore, $\varrho^0 = \varrho^{\mathrm{B}}$. It is straightforward to show that firm value is single-peaked around ϱ^{B}. □

Proof of Lemma 4

First, the logic of "the informed shareholder must vote according to the advice of the PA in equilibrium" still holds here. In other words, the shareholder follows the PA's recommendation as long as she decides to purchase it in equilibrium.

Second, to show the shareholder not purchasing the recommendation cannot constitute an equilibrium, it is useful to compute the information value of the PA's recommendation. If the shareholder is informed, the payoff of voting for the proposal is

$$\begin{aligned}&\Pr(\theta=1|r=1)-\gamma\Pr(\theta=0|r=1)\\&=\frac{\Pr(r=1|\theta=1)\Pr(\theta=1)-\gamma\Pr(r=1|\theta=0)\Pr(\theta=0)}{\Pr(r=1)}\\&=\frac{A_1}{\pi_1\mu+(1-\pi_0)(1-\mu)},\end{aligned}$$

which shares the same form as equation (A1). Similarly, we obtain

$$\Pr(\theta=1|r=0)-\gamma\Pr(\theta=0|r=0)=\frac{-A_0}{(1-\pi_1)\mu+\pi_0(1-\mu)}.$$

The argument for $A_1, A_0>0$ in equilibrium is the same as that in the proof of Lemma 1.

If $\gamma\le\frac{\mu}{(1-\mu)}$, the shareholder would vote for the proposal if she had not purchased the recommendation (Assumption A2 kicks in when the equality holds). Under this case, only $r=0$ is valuable as it can swing the shareholder's action while $r=1$ cannot. Therefore, the information value is

$$V(\rho)=V(\rho|r=0)\Pr(r=0)=\left(0-\frac{-A_0}{(1-\pi_1)\mu+\pi_0(1-\mu)}\right)\Pr(r=0)=A_0$$

if $\gamma\le\frac{\mu}{(1-\mu)}$. Similarly, if $\gamma>\frac{\mu}{(1-\mu)}$ the information value is

$$V(\rho)=V(\rho|r=1)\Pr(r=1)=\frac{A_1}{\pi_1\mu+(1-\pi_0)(1-\mu)}\Pr(r=1)=A_1.$$

Because the voting recommendation has a positive value, the shareholder would purchase it if its price is low enough. Therefore, to maximize its profits, the PA would not set $f=0$ in equilibrium. To prove that the shareholder purchases the PA's recommendation with probability one in equilibrium, we assume the shareholder purchases the PA's recommendation with probability $q \in (0,1)$. Note that the PA's profit is $qf + \Pr(r=1)\varphi$ in this hypothetical case. For the shareholder to use a strictly mixed strategy, she must be indifferent, that is, $f = V(\varrho)$. However, the PA can deviate to setting $f = \frac{1+q}{2}V(\varrho) < V(\varrho)$. By doing so, the PA can induce the shareholder to purchase its recommendation for sure and realize a profit of $\frac{1+q}{2}V(\varrho) + \Pr(r=1)\varphi$, which is strictly larger than $qV(\varrho) + \Pr(r=1)\varphi$. In short, the PA has a profitable deviation by slightly decreasing the charged price if the shareholder does not purchase its recommendation for sure.

If $\varphi = 0$, the PA's problem is straightforward to write down:

$$\max_{\varrho} V(\varrho) = \begin{cases} A_1 & \text{if } \gamma > \frac{\mu}{(1-\mu)}, \\ A_0 & \text{if } \gamma \leq \frac{\mu}{(1-\mu)}. \end{cases}$$

Because both A_1 and A_0 are maximized at $\varrho = \hat{\varrho}$, the PA sets the recommendation threshold to $\hat{\varrho}$.

The third part is obvious because essentially the implementation of the proposal is solely based on the PA's unbiased recommendation. □

Proof of Proposition 2

To begin with, we write shareholders' expected subjective utility (gross of the potential fee paid for the voting recommendations) as follows:

$$\mathcal{U}(q,\varrho) = \mu \Pr(d=1|\theta=1) - \gamma(1-\mu)\Pr(d=1|\theta=0). \tag{A26}$$

In the remaining of the proof, we simply call this shareholders' subjective utility. We prove the following auxiliary lemma.

Lemma A4. *Assume the voting-stage equilibrium and the information-purchase equilibrium. When $\varphi = 0$, shareholders' subjective expected utility (gross of the potential fee paid for the* *voting recommendations)* $\mathcal{U}(q, \varrho)$ *is strictly increasing with respect to* q *for all* ϱ*, and single-peaked around* $\hat{\varrho}$ *with respect to* ϱ *for all* q*, where* $\hat{\varrho}$ *is given by equation* (15)*.*

Proof. Before deriving, we first define the function $P(x, N, k)$ as the probability that the proposal gets k votes out of N when each shareholder independently votes for the proposal with probability x:

$$P(x, N, k) \equiv \binom{N}{k} x^k (1-x)^{N-k}, \tag{A27}$$

where $\binom{N}{k} = N!/(k!(N-k)!)$ is the binomial coefficient.

Let ω_1 be the probability with which a randomly chosen shareholder votes for the proposal conditional on the PA sending $r = 1$, and ω_0 be the probability of voting in favor of the proposal when the PA sends $r = 0$. That is,

$$\omega_1 \equiv \Pr(v = 1 | r = 1) = q + (1-q)\tau \quad \text{and} \quad \omega_0 \equiv \Pr(v = 1 | r = 0) = (1-q)\tau. \tag{A28}$$

Therefore, we have the following:

$$\Pr(d = 1 | r = 1) = \sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k), \tag{A29}$$

$$\Pr(d = 1 | r = 0) = \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k). \tag{A30}$$

Provided that we have two states and the PA can send two types of voting recommendations, four possible cases can be considered: (1) $\theta = 1$ and $r = 1$; (2) $\theta = 1$ and $r = 0$; (3) $\theta = 0$ and $r = 1$; and (4) $\theta = 0$ and $r = 0$. Thus, with the substitution of equations (3), (4), (A29) and (A30), equation (A26) can be rearranged as follows:

$$\begin{aligned} \mathcal{U} = \mu\pi_1 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) + \mu(1-\pi_1) \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k) \\ - \gamma(1-\mu)(1-\pi_0) \sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) - \gamma(1-\mu)\pi_0 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k), \end{aligned} \tag{A31}$$

Note that $\Pr(d|\theta, r) = \Pr(d|r)$ because the state is not observed by shareholders.

Using the fact that

$$\sum_{k=\frac{N+1}{2}}^{N} P(x, N, k) = \frac{B\left(x; \frac{N+1}{2}, \frac{N+1}{2}\right)}{B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)},$$

equation (A31) can be simplified to

$$\begin{aligned} B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\mathcal{U} = {} & [\mu\pi_1 - \gamma(1-\mu)(1-\pi_0)] B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) \\ & + [\mu(1-\pi_1) - \gamma(1-\mu)\pi_0] B\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right), \end{aligned} \quad \text{(A32)}$$

where $B(x; a, b) = \int_0^x t^{a-1}(1-t)^{b-1}\mathrm{d}t$ is the incomplete beta function and $B(a, b)$ is the beta function. Taking the derivative of the RHS of equation (A32) with respect to ϱ, we have the following:

$$\begin{aligned} & \left(\mu\frac{\partial \pi_1}{\partial \varrho} + \gamma(1-\mu)\frac{\partial \pi_0}{\partial \varrho}\right)\left(B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) - B\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right)\right) \\ & + (1-q)\frac{\partial \tau}{\partial \varrho}\left[\begin{array}{c} (\mu\pi_1 - \gamma(1-\mu)(1-\pi_0)) B_{\omega_1}\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) \\ + (\mu(1-\pi_1) - \gamma(1-\mu)\pi_0) B_{\omega_0}\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right) \end{array}\right]. \end{aligned} \quad \text{(A33)}$$

One can easily verify that $B(x; a, b)$ is strictly increasing with respect to x by calculating

$$B_x\left(x; \frac{N+1}{2}, \frac{N+1}{2}\right) = (x(1-x))^{\frac{N-1}{2}} > 0. \quad \text{(A34)}$$

Thus, we have

$$\begin{aligned} & B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) > B\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right), \\ & B_{\omega_1}\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) = (q + (1-q)\tau)^{\frac{N-1}{2}}((1-q)(1-\tau))^{\frac{N-1}{2}}, \end{aligned}$$

and

$$B_{\omega_0}\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right) = ((1-q)\tau)^{\frac{N-1}{2}}(1-(1-q)(1-\tau))^{\frac{N-1}{2}}.$$

Using the indifference condition for uninformed shareholders in equation (A5), one can conclude that

$$(\mu\pi_1 - \gamma(1-\mu)(1-\pi_0))\, B_{\omega_1}\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) + \\ (\mu(1-\pi_1) - \gamma(1-\mu)\pi_0)\, B_{\omega_0}\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right) = 0. \quad \text{(A35)}$$

Therefore, equation (A33) can be rearranged as

$$B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\frac{\partial \mathcal{U}}{\partial \varrho} = (-\mu h_1(\varrho) + \gamma(1-\mu)h_0(\varrho)) \\ \times \left(B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) - B\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right)\right).$$

One can easily check that when $\varrho < \hat{\varrho}$, $\frac{\partial \mathcal{U}}{\partial \varrho} > 0$, whereas when $\varrho > \hat{\varrho}$, $\frac{\partial \mathcal{U}}{\partial \varrho} < 0$. So, $\mathcal{U}$ is single-peaked at $\varrho = \hat{\varrho}$ with respect to ϱ for all q.

Next, taking the derivative of the RHS of equation (A32) with respect to q, we have the following:

$$(\mu\pi_1 - \gamma(1-\mu)(1-\pi_0))\, B_{\omega_1}\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) + \\ \left((1-q)\frac{\partial \tau}{\partial q} - \tau\right)\left[\begin{array}{c} (\mu\pi_1 - \gamma(1-\mu)(1-\pi_0))\, B_{\omega_1}\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) \\ + (\mu(1-\pi_1) - \gamma(1-\mu)\pi_0)\, B_{\omega_0}\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right) \end{array}\right]. \quad \text{(A36)}$$

Similarly based on equation (A35), we have

$$B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\frac{\partial \mathcal{U}}{\partial q} = (\mu\pi_1 - \gamma(1-\mu)(1-\pi_0))\, B_{\omega_1}\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) > 0.$$

Therefore, $\mathcal{U}$ is monotonically increasing in q. □

We now prove the equilibrium in the benchmark. When $\gamma = 1$ and $\mu = \mu_0$, equations (18) and (A26) are equivalent; that is, $\mathcal{U} = \Psi$. Therefore, according to Proposition 1, Definition 2, and Lemma A4, for any given q, the equilibrium threshold $\varrho^* = \hat{\varrho}$ maximizes the expected firm value. Further, because $\frac{\partial \Psi}{\partial q} = \frac{\partial \mathcal{U}}{\partial q} > 0$, at $q = q^* < 1$, firm value may be improved by increasing q. $\square$

Proof of Corollary 1

To prove that $\Psi^{\mathrm{B}} < \Psi^0$, we only need to prove that firm value equals to Ψ^0 when $q = 1$ and $\varrho = \varrho^{\mathrm{B}}$ in the benchmark economy (i.e., $\Psi(1, \varrho^{\mathrm{B}}; 1, \mu_0, 0) = \Psi^0$). This is because based on Lemma A4, we know that firm value (which is the same as $\mathcal{U}$ in the benchmark) is increasing with respect to q, so the equilibrium firm value in the benchmark economy must be lower than the case in which $q = 1$, that is, $\Psi(q^{\mathrm{B}}, \varrho^{\mathrm{B}}; 1, \mu_0, 0) < \Psi(1, \varrho^{\mathrm{B}}; 1, \mu_0, 0) = \Psi^0$.

We now show that $\Psi(1, \varrho^{\mathrm{B}}; 1, \mu_0, 0) = \Psi^0$. In the benchmark economy, when $q = 1$, we have $\Pr(d = 1|r = 1) = 1$ and $\Pr(d = 1|r = 0) = 0$ because all shareholders purchase the PA's recommendations and every shareholder follows the recommendation as analysed in the proof of Lemma A1. So firm value, which is equivalent to shareholders' subjective utility (gross of the potential fee paid for the voting recommendations) in equation (A31) can be simplified to $\Psi(1, \varrho^{\mathrm{B}}; 1, \mu_0, 0) = \mu_0\pi_1 + (1 - \mu_0)(1 - \pi_0)$, where π_1 and π_0 are given by equations (3) and (4) at $\varrho = \varrho^{\mathrm{B}}$. Further, based on the proof of Lemma 3, $\Psi^0 = \mu_0 \left(1 - H_1(\varrho^{\mathrm{B}})\right) - (1 - \mu_0) \left(1 - H_0(\varrho^{\mathrm{B}})\right)$, which is equal to $\Psi(1, \varrho^{\mathrm{B}}; 1, \mu_0, 0)$ because $\pi_1 = 1 - H_1(\varrho^{\mathrm{B}})$ and $\pi_0 = H_0(\varrho^{\mathrm{B}})$. $\square$

Proof of Corollary 2

When $\varphi \neq 0, \gamma = 1$, and $\mu_0 = \mu$, firm value is equal to shareholders' subjective utility (gross of the potential fee paid for the voting recommendations): $\Psi = \mathcal{U}$. Based on equations (15) and (17), $\hat{\varrho} = \varrho^{\mathrm{B}}$. According to Proposition 1, when $\varphi > (<) 0$, $\varrho^* < (>) \, \hat{\varrho} = \varrho^{\mathrm{B}}$ and $q^* \geq q^*(\varrho^{\mathrm{B}})$. Next, we show that when $\varphi = 0$, $\frac{\partial \Psi}{\partial \varphi} = 0$. Firm value is

$$B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\Psi = [\mu_0\pi_1 - (1 - \mu_0)(1 - \pi_0)]B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) + [\mu_0(1 - \pi_1) - (1 - \mu_0)\pi_0]B\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right). \quad \text{(A37)}$$

Because φ only affects firm value Ψ through q and ϱ, we have

$$\frac{\partial \Psi}{\partial \varphi} = \frac{\partial \Psi}{\partial \varrho}\frac{\partial \varrho}{\partial \varphi} + \frac{\partial \Psi}{\partial q}\frac{\partial q}{\partial M}\frac{\partial M}{\partial \varrho}\frac{\partial \varrho}{\partial \varphi}.$$

When $\varphi = 0$, based on Lemma A4, we have $\frac{\partial \Psi}{\partial \varrho} = 0$ and based on Lemma A2, we have $\frac{\partial M}{\partial \varrho} = 0$. Therefore, at $\varphi = 0$, $\frac{\partial \Psi}{\partial \varphi} = 0$.

We can analytically prove that Ψ is single-peaked around $\varphi = 0$ when $\mu_0 = \frac{1}{2}$. When $\mu_0 = \frac{1}{2}$, according to Proposition 1, $q^* = \frac{1}{\sqrt{N}}$. Based on Lemma A4, firm value (which is equal to $\mathcal{U}$) is single peaked at ϱ^{B}. For any $\varphi \neq 0$, we know that $\varrho^* \neq \varrho^{\mathrm{B}}$, and thus firm value must be lower than Ψ^{B}. □

Proof of Propositions 3 and 4

When $\varphi = 0$, ϱ^* satisfies equation (15), whereas ϱ^{B} satisfies equation (17). Therefore, when $\gamma \neq 1$ and $\mu = \mu_0$, one can easily obtain that if $\gamma > (<)1$, $\varrho^* > (<)\varrho^{\mathrm{B}}$. Similarly, when $\gamma = 1$ and $\mu \neq \mu_0$, if $\mu > (<)\mu_0$, $\varrho^* < (>)\varrho^{\mathrm{B}}$.

The effect on q^*. Next, we prove that the effect of the parameter change on q^*. Consider the case $\mu = \mu_0$ and examine the effect of γ. Based on Equations (9) and (16), taking the total derivative of q with respect to γ yields

$$\frac{\partial q}{\partial \gamma} = \frac{\partial q}{\partial M}\left(\frac{\partial M}{\partial \gamma} + \frac{\partial M}{\partial \varrho}\frac{\partial \varrho}{\partial \gamma}\right) = \frac{\partial q}{\partial M}\frac{\partial M}{\partial \gamma}.$$

The second equality follows because $\varrho^* = \hat{\varrho}$ (Proposition 1) and $\frac{\partial M}{\partial \varrho} = 0$ at $\varrho = \hat{\varrho}$ (equation (A11)).

Next we discuss the sign of $\frac{\partial q}{\partial \gamma}$. We can derive that

$$\frac{\partial M}{\partial \gamma} = \frac{(1-\mu)\mu(\pi_0 + \pi_1 - 1)}{(\gamma(1-\mu)(1-\pi_0) - \mu\pi_1)^2} > 0.$$

The inequality holds because $\pi_1 + \pi_0 - 1 = H_0(\varrho) - H_1(\varrho) > 0$ for all ϱ. Recall that $h_1(\varrho)$ is assumed to likelihood dominate $h_0(\varrho)$, then $h_1(\varrho)$ first-order stochastically dominates $h_0(\varrho)$, that is, $H_1(\varrho) < H_0(\varrho)$. So $\frac{\partial q}{\partial \gamma}$ at $\gamma = 1$ has the same sign as $\frac{\partial q}{\partial M}$, which has the same sign as

$M-1$ according to equation (A25). Therefore, at $\gamma=1$, $\frac{\partial q}{\partial \gamma} > (<) 0$ when $\gamma > (<) \frac{\mu}{1-\mu}$, or equivalently, when $\mu_0 < (>)\, 0.5$ due to $\mu = \mu_0$. Finally, at $\gamma = 1$, when $\mu_0 = 0.5$, $\frac{\partial q}{\partial \gamma} = 0$. Further, if $\gamma < 1$, $M < 1$ and $\frac{\partial q}{\partial M} < 0$, so $\frac{\partial q}{\partial \gamma} < 0$. And if $\gamma > 1$, $M > 1$ and $\frac{\partial q}{\partial M} > 0$, so $\frac{\partial q}{\partial \gamma} > 0$. Therefore, when $\mu_0 = 0.5$, q^* reaches the local minimum point at $\gamma = 1$.

The effect of μ on q^* can be similarly analyzed and we skip it here.

The effect on firm value Ψ^*. Finally, we discuss the effect of the parameter change on firm value Ψ^* under limiting conditions. First, consider the effect of γ at $\gamma = 1$ (note that $\mu = \mu_0$). Similar to the calculation of $\mathcal{U}$ in the proof of Proposition A4, firm value can be computed as follows:

$$B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\Psi = [\mu_0\pi_1 - (1-\mu_0)(1-\pi_0)]B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) + [\mu_0(1-\pi_1) - (1-\mu_0)\pi_0]B\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right). \tag{A38}$$

Taking the total derivative of Ψ with respect to γ we obtain that

$$\begin{aligned} &B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\frac{\partial \Psi}{\partial \gamma} \\ &= [\mu_0\pi_1 - (1-\mu_0)(1-\pi_0)]B_{\omega_1}\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right)\left(\frac{\partial \omega_1}{\partial q}\frac{\partial q}{\partial \gamma} + \frac{\partial \omega_1}{\partial \varrho}\frac{\partial \varrho}{\partial \gamma} + \frac{\partial \omega_1}{\partial \gamma}\right) \\ &+ [\mu_0(1-\pi_1) - (1-\mu_0)\pi_0]B_{\omega_0}\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right)\left(\frac{\partial \omega_0}{\partial q}\frac{\partial q}{\partial \gamma} + \frac{\partial \omega_0}{\partial \varrho}\frac{\partial \varrho}{\partial \gamma} + \frac{\partial \omega_0}{\partial \gamma}\right) \\ &+ ((1-\mu_0)h_0(\varrho) - \mu_0 h_1(\varrho))\left(B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) - B\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right)\right), \end{aligned} \tag{A39}$$

where based on equations (8), (9) and (A28),

$$\frac{\partial \omega_1}{\partial q}\frac{\partial q}{\partial \gamma} + \frac{\partial \omega_1}{\partial \varrho}\frac{\partial \varrho}{\partial \gamma} + \frac{\partial \omega_1}{\partial \gamma} = \left(1 - \tau + (1-q)\frac{\partial \tau}{\partial q}\right)\frac{\partial q}{\partial \gamma} + (1-q)\frac{\partial \tau}{\partial M}\frac{\partial M}{\partial \varrho}\frac{\partial \varrho}{\partial \gamma} + (1-q)\frac{\partial \tau}{\partial M}\frac{\partial M}{\partial \gamma}$$

and

$$\frac{\partial \omega_0}{\partial q}\frac{\partial q}{\partial \gamma}+\frac{\partial \omega_0}{\partial \varrho}\frac{\partial \varrho}{\partial \gamma}+\frac{\partial \omega_0}{\partial \gamma}=$$
$$\left(-\tau+(1-q)\frac{\partial \tau}{\partial q}\right)\frac{\partial q}{\partial \gamma}+(1-q)\frac{\partial \tau}{\partial M}\frac{\partial M}{\partial \varrho}\frac{\partial \varrho}{\partial \gamma}+(1-q)\frac{\partial \tau}{\partial M}\frac{\partial M}{\partial \gamma}.$$

Therefore, equation (A39) can be simplified as follows:

$$\begin{aligned}
&B\left(\frac{N+1}{2},\frac{N+1}{2}\right)\frac{\partial \Psi}{\partial \gamma}\\
&\quad=\Upsilon\times\left(\left(-\tau+(1-q)\frac{\partial \tau}{\partial q}\right)\frac{\partial q}{\partial \gamma}+(1-q)\frac{\partial \tau}{\partial M}\frac{\partial M}{\partial \varrho}\frac{\partial \varrho}{\partial \gamma}+(1-q)\frac{\partial \tau}{\partial M}\frac{\partial M}{\partial \gamma}\right)\\
&\qquad\quad+\left[\mu_0\pi_1-(1-\mu_0)(1-\pi_0)\right]B_{\omega_1}\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)\frac{\partial q}{\partial \gamma}\\
&+\left((1-\mu_0)h_0(\varrho)-\mu_0 h_1(\varrho)\right)\left(B\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)-B\left(\omega_0;\frac{N+1}{2},\frac{N+1}{2}\right)\right),
\end{aligned}\tag{A40}$$

where

$$\begin{aligned}
\Upsilon=&\left(\mu_0\pi_1-(1-\mu_0)(1-\pi_0)\right)B_{\omega_1}\left(\omega_1,\frac{N+1}{2},\frac{N+1}{2}\right)\\
&+\left(\mu_0(1-\pi_1)-(1-\mu_0)\pi_0\right)B_{\omega_0}\left(\omega_0,\frac{N+1}{2},\frac{N+1}{2}\right).
\end{aligned}\tag{A41}$$

Notice that when $\mu=\mu_0$, based on equation (A35) we have

$$\begin{aligned}
&\left(\mu_0\pi_1-\gamma(1-\mu_0)(1-\pi_0)\right)B_{\omega_1}\left(\omega_1,\frac{N+1}{2},\frac{N+1}{2}\right)\\
&\quad+\left(\mu_0(1-\pi_1)-\gamma(1-\mu_0)\pi_0\right)B_{\omega_0}\left(\omega_0,\frac{N+1}{2},\frac{N+1}{2}\right)=0.
\end{aligned}\tag{A42}$$

Thus, when $\gamma=1$, we have $\Upsilon=0$. Further, when $\mu=\mu_0$, at $\gamma=1$, based on equation (15) we have $(1-\mu_0)h_0(\varrho)-\mu_0h_1(\varrho)=0$ in equilibrium; based on equation (6), we have $\mu_0\pi_1-(1-\mu_0)(1-\pi_0)>0$ in equilibrium; based on equation (A34), $B_{\omega_1}\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)>0$. So, the sign of $\frac{\partial \Psi}{\partial \gamma}$ is the same as $\frac{\partial q}{\partial \gamma}$, whose property has been analyzed above. Therefore, at $\gamma=1$, $\frac{\partial \Psi}{\partial \gamma}>(<)0$ when $\mu_0<(>)\frac{1}{2}$. Finally, we discuss the case in which $\mu_0=\frac{1}{2}$. When

$\mu_0 = \frac{1}{2}$, $\frac{\partial q}{\partial \gamma} = 0$ and hence $\frac{\partial \Psi}{\partial \gamma} = 0$.

We need further determine whether Ψ is single-peaked at $\gamma = 1$. Specifically, when $\mu_0 = \frac{1}{2}$, based on equation (16) we know that $q^* = \frac{1}{\sqrt{N}}$, $M = 1$, and $\tau = \frac{1}{2}$, so equation (A38) can be simplified as follows:

$$\begin{aligned} B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\Psi &= \frac{1}{2}\left(\pi_1 + \pi_0 - 1\right) \\ &\times \left(B\left(\frac{1}{2}\left(1 + \frac{1}{\sqrt{N}}\right); \frac{N+1}{2}, \frac{N+1}{2}\right) - B\left(\frac{1}{2}\left(1 - \frac{1}{\sqrt{N}}\right); \frac{N+1}{2}, \frac{N+1}{2}\right)\right). \end{aligned} \tag{A43}$$

So the sign of Ψ is determined by $\pi_1 + \pi_0 - 1$, whose dependence on γ is given as follows:

$$\frac{\partial\left(\pi_1 + \pi_0 - 1\right)}{\partial \gamma} = \left(h_0(\varrho) - h_1(\varrho)\right) \times \frac{\partial \varrho}{\partial \gamma} < (>)\, 0$$

when $\gamma > (<)\, 1$. Therefore, Ψ is single-peaked at $\gamma = 1$.

The effect of μ on Ψ^* is similarly analyzed and we skip it here. $\square$

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