Secret and Overt Information Acquisition in Financial Markets *

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Abstract

We study the observability of investors' information-acquisition activities in financial markets. Improving observability leads to two strategic effects on information acquisition: (i) The pricing effect, which arises from interactions between investors and the market maker and can encourage or discourage information acquisition; and (ii) the competition effect, which concerns interactions amongst investors and always encourages information acquisition. We apply our theory to study voluntary and mandatory disclosures of corporate site visits. When the competition effect dominates, investors voluntarily disclose their visits. When the pricing effect dominates, mandatory disclosure is effective. Our analysis sheds novel light on Regulation Fair Disclosure.

Keywords: Information acquisition, observability, corporate site visits, disclosure, Regulation Fair Disclosure
JEL: D82, G14, G18

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Information acquisition is a key decision for many investors who participate in financial markets. One important but understudied characteristic of this activity is its (un)observability.[1] More often than not, information acquisition is unobservable. The coverage and depth of a fund's in-house research, for instance, are generally unknown to the public, and sophisticated investors often go to great lengths to conceal and erase their footprints, thus keeping their information acquisition secretive.

On the other hand, regulatory or economic forces can cause information-acquisition observability to increase under various contexts. For instance, following the promulgation of Regulation Fair Disclosure (Reg FD), the list of participants in conference calls between management and investors has become more observable to the general public. In its own Reg FD, China's Shenzhen Stock Exchange (SZSE) now requires all listed firms to disclose their investors' site visits within two trading days of the event, which formally mandates the disclosure of these private meetings. Meanwhile, some funds are often observed to voluntarily disclose their site visits, which again implies more transparent information acquisition.

In this paper, we develop a parsimonious model that accommodates different degrees of information-acquisition observability. Our model aims to answer the following research questions: (i) How does the observability of information acquisition affect investors' information-acquisition and trading behaviors as well as market quality? (ii) What incentives do investors have to voluntarily disclose their information-acquisition activities? (iii) When is the mandatory disclosure of such activities effective in changing investors' information acquisition, and what are the effects and consequences of mandatory disclosure? Addressing these questions not only offers theoretical insights on the implications of changing observability surrounding information acquisition but also helps us further dissect empirical regularities and policy debates.

Our model builds on Kyle's (1985) canonical, static, single-asset framework. As standard, multiple strategic, risk-neutral investors and a mass of noise traders submit market orders to

[1] Many existing studies assume that investors' information acquisition is perfectly observable (e.g., Admati and Pfleiderer, 1988; Holden and Subrahmanyam, 1992) while only a few, such as Banerjee and Breon-Drish (2020) and Rüdiger and Vigier (2020), have explicitly considered unobservable information acquisition.

a risk-neutral market maker who sets a risky asset's price as its expected value conditional on the aggregate order flow. Strategic investors spend costs to acquire diverse private information regarding the asset's payoff. The innovation of our model is its introduction of a communication technology that sends general public messages, which may contain information regarding the precision levels of private information acquired by the investors.[2] The informativeness of these messages determines the degrees of observability surrounding the investors' information-acquisition activities.

We conduct our analysis in two sections. In Section 2, we compare two abstract benchmark settings to illustrate the difference between overt and secret information acquisition, which helps us develop clean intuitions for subsequent applications. Under the overt benchmark setting, messages perfectly reveal investors' information precision so that their information-acquisition activities are completely observable. Under the secret benchmark setting, messages do not directly reveal any information regarding investors' information precision, and thus, their activities are completely unobservable. By comparing these settings, we find that increasing the observability of information acquisition has two strategic effects on equilibrium outcomes: (1) The "pricing effect," which results from interactions between investors and the market maker, and (2) the "competition effect," which results from interactions among investors.

In a monopoly economy, only the pricing effect operates, and it deters information acquisition by an overt monopolistic investor relative to a secret monopolistic investor. Specifically, if an overt monopolistic investor increases her information precision, the market maker recognizes the more informed investor and adjusts the pricing schedule to better manage adverse-selection risk, thereby reducing the investor's profits and, in turn, her information-acquisition incentives. As a result, in equilibrium, an overt monopolistic investor acquires less information than a secret monopolistic investor, and lower information production causes the overt market to experience

[2] This relates our paper to studies on uncertainty surrounding strategic investors' informedness (e.g., Chakraborty and Yılmaz, 2004; Hughes and Pae, 2004; Bagnoli and Watts, 2007; Altı, Kaniel, and Yoeli, 2012; Banerjee and Green, 2015; Back, Crotty, and Li, 2018; Dai, Wang, and Yang, 2019; Peress and Schmidt, 2019). Although the literature assumes this uncertainty comes from nature, our model has no such uncertainty because, in equilibrium, market participants hold correct beliefs about strategic investors' decisions to acquire information.

lower efficiency as measured by the posterior precision of the fundamental conditional on asset prices. Given the less severe adverse-selection problem faced by the market maker, the overt market is also associated with higher market liquidity, measured by the inverse of noise trading's price impact. This pricing effect connects our paper to studies on information sales (e.g., Admati and Pfleiderer, 1986, 1990), which have found that a profit-maximizing information seller can optimally add noise to the sold information to reduce price informativeness and thus curb information leakage. Likewise, the overt monopolistic investor in our setting also favors lower price informativeness; however, instead of adding noise, she achieves this by committing to less information production, and her benefit comes from the improved market liquidity rather than from the reduced information leakage.

In an oligopoly economy, both the pricing and competition effects are active. The competition effect works through interactions among investors: Overtly acquiring information by an investor decreases incentives for other investors to trade aggressively on their private information. Since the aggressive trading of others is harmful to the initial investor's profits, the decrease in their trading aggressiveness gives the initial investor an additional incentive to acquire information. As a result, the competition effect encourages more information production when information is acquired overtly rather than secretly. The mechanism bears some similarity to what Hauk and Hurkens (2001) find in Cournot product markets. At the most basic level, the competition effect is reminiscent of the Stackelberg leadership effect, in which a leader strategically chooses high production to force followers to reduce production. Unlike in the monopoly economy, the pricing effect is ambiguous under this oligopoly economy. When the number of investors is small or their information quality is low, the pricing effect works as it would under the monopoly economy but in a contrasting way to the competition effect (i.e., it discourages information production under overt information acquisition). Only when the pricing effect prevails can the overt market continue to produce less precise information in equilibrium, leading to lower market efficiency and higher liquidity.

In Section 3, we choose corporate site visits as one specific context through which we demonstrate how the pricing and competition effects identified in Section 2 help clarify empirical regularities and regulatory debates. Site visits refer to field trips that investors take to company headquarters and production facilities. The literature documents that, by physically interacting with company employees, investors can collect useful information that guides their portfolio decisions (e.g., Solomon and Soltes, 2015; Cheng et al., 2019; Chen et al., 2022; Cicero et al., 2021). Section 3 discusses how site visits are disclosed voluntarily or mandatorily in practice and maps them to our model. We argue that, if an investor is publicly observed to have conducted a site visit (which can be achieved either by voluntary or mandatory disclosure), then other agents know that the investor's precision level is bounded below by a threshold. If the investor is observed not to have conducted a visit (which can be achieved only by mandatory disclosure), then the public knows that the investor's precision level is bounded above by the same threshold.

In a monopoly economy, the monopolistic investor does not voluntarily disclose and effectively engages in secret information acquisition. This result is driven by the pricing effect since voluntarily disclosing a conducted visit reveals to the market maker a lower bound of the investor's precision level, which, if anything, can worsen the market maker's adverse-selection concern and reduce market liquidity, thereby harming the investor. Under mandatory disclosure, the investor can reveal to the market maker either the lower bound (if she conducts a visit) or upper bound (if she conducts no visits) of her precision level. The upper bound can weaken the market maker's adverse-selection concern, which, again, benefits the investor through the pricing effect. When this effect is strong, the monopolistic investor can refuse site visits that she would've otherwise conducted under voluntary disclosure.

In an oligopoly economy, the competition effect kicks in. Recall that the competition effect suggests that an investor wants her rivals' belief about her information precision to be the highest possible to deter them from trading aggressively. So, this effect encourages an investor to conduct a site visit and disclose the event to inform her rivals of her precision level being above a threshold. When there are many rivals and the information-acquisition cost is low, investors end up with precise information and compete aggressively, making the competition effect particularly strong.

In this case, the mandatory and voluntary disclosure settings always share the same equilibrium in which investors conduct site visits, which they would not have conducted in the absence of a communication technology, and disclose. By contrast, when the number of investors is small or the information-acquisition cost is high, an oligopoly economy resembles a monopoly economy, and the equilibrium differs across mandatory and voluntary disclosures.

Our analysis of site visits makes empirical predictions and sheds light on regulatory debates. Given that voluntary disclosure of site visits is driven by the competition effect, our analysis predicts that financial institutions are more likely to disclose their otherwise secret site visits when the competition effect is strong, which happens when the number of investors trading the same asset is large and the cost of acquiring information is low. Comparing the voluntary disclosure setting with the mandatory disclosure setting, our model suggests that mandatory disclosure is effective when the pricing effect dominates, which requires the number of investors to be small or the cost of acquiring information to be high. In addition, effective mandatory disclosure causes investors to acquire less precise information, thereby increasing their payoffs and market liquidity but decreasing market efficiency.

There continues an active regulatory debate on whether to mandate the disclosure of the occurrence and content of corporate site visits (e.g., Ng and Troianovski, 2015; Solomon and Soltes, 2015; Bengtzen, 2017; Soltes, 2018; Bowen et al., 2020). Our results on the effectiveness and consequences of mandatory disclosure can be useful for policymakers weighing the relevant costs and benefits of this practice. So far, SZSE is the only regulatory jurisdiction to demand the disclosure of site visits. If other jurisdictions hope to implement similar regulations within their own contexts, they must understand when such disclosure is effective and what consequences it can induce. Our results offer guidance on these topics.

Finally, we use our analysis to shed new light on Reg FD, which prevents the selective disclosure of material non-public information, and other contexts, such as social networks and geography of investors, which involve changing information-acquisition observability. We argue that, in these contexts, investors' information-acquisition activities become more observable, which is akin to a switch from the secret to overt benchmark in Section 2 or from voluntary to mandatory disclosure in Section 3. Accordingly, our theory predicts that, in these contexts, information production and market efficiency decrease but market liquidity increases for firms with a small number of investors or a high cost of information acquisition.

1 A Model of Secret and Overt Information Acquisition in Financial Markets

Setup. We consider a Kyle-type model (Kyle, 1985) with three dates ($t = 0, 1, 2$). Figure 1 describes the timeline of the economy. There is a single risky asset with a date-2 liquidation value $\tilde{v}$, where $\tilde{v} \sim N(0, 1)$.[3] The financial market operates on date 1 and is populated by three groups of agents: One market maker, noise traders, and J rational investors, where $J \geq 1$ is a positive integer. The market maker and rational investors are risk neutral. As typical, the market maker provides liquidity by setting the asset's price according to the weak market-efficiency rule. Noise traders and rational investors demand liquidity; noise traders submit exogenous random market orders to meet their unmodeled liquidity needs while rational investors trade on private information to earn profits. On date 0, rational investors engage in information acquisition, and a communication technology may reveal some messages regarding investors' information-acquisition activities to the general public. This technology is our setup's primary innovative element, and we will discuss it shortly in detail.

Information Acquisition. On $t = 0$, J investors simultaneously make their information-acquisition decisions. Consider investor $j \in \{1, ..., J\}$. Prior to trading on date 1, she can observe a private signal in the form: $\tilde{y}_j = \tilde{v} + \tilde{e}_j$, where $\tilde{e}_j \sim N(0, h_j^{-1})$ and $\tilde{v}$ and $\tilde{e}_j$ are mutually independent. Investor j will use signal $\tilde{y}_j$ to guide her trade. The more precise the signal, the

[3] The normalization that $\tilde{v}$ has a zero mean and a unit standard deviation is without loss of generality. Instead, if we assume $\tilde{v} \sim N(\bar{v}, \sigma_v^2)$ (with $\bar{v} \in \mathbb{R}$ and $\sigma_v > 0$), then all of our results still hold as long as we reinterpret information precision as signal-to-noise ratio.

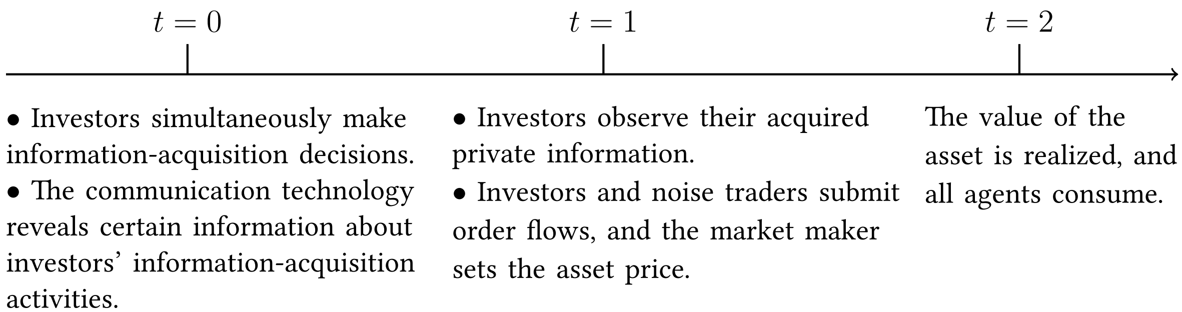


Figure 1: Timeline

higher the trading profits she expects to obtain. Precision h_j can be acquired according to an increasing cost function $C(h_j)$. In our analysis, we consider a linear cost function: $C(h_j) = c \cdot h_j$, where c is a positive constant. Commonly adopted in the literature (e.g., Verrecchia, 1982; Kim and Verrecchia, 1991; Hauk and Hurkens, 2001; Myatt and Wallace, 2012), this linear cost function can be justified as discrete sampling with a constant cost per observation. Investor j chooses precision h_j to maximize her expected trading profits net of the information-acquisition cost.

Communication Technology. On $t = 0$, there exists a communication technology that can potentially reveal investors' information-acquisition activities to the public. Specifically, after investor j acquires information, all active agents (i.e., investors and the market maker) observe a message m_j that is generated according to a message function $m_j(h_j)$. The message m_j may contain information about investor j's information precision h_j so that agents can use m_j to update their beliefs about h_j. In this section, we introduce these message functions on an abstract level and are agnostic to what the messages are and where the message functions come from. These abstractions allow us to use a single setting to explore a variety of relevant contexts.

Two special cases of message functions deserve particular attention. In one case, the message m_j perfectly reveals the precision h_j of the private signal acquired by investor j. Without loss of generality, we denote the message as $m_j = h_j$. In this case, investor j engages in *overt* information acquisition. In the second case, m_j does not directly reveal any information about investor j's h_j, and we denote the message as $m_j = \varnothing$. In this case, the investor *secretly* acquires information. In Section 2, we explore two benchmark economies, one with all overt investors (i.e., the overt benchmark economy) and another with all secret investors (i.e., the secret benchmark economy). We compare these two benchmark economies to transparently develop intuitions for the subsequent application in Section 3.

In Section 3, we consider the concrete application of corporate site visits through which financial institutions visit companies to acquire private information. The observability of those visits can be affected by voluntary or mandatory disclosure. Under voluntary disclosure, a financial institution can decide whether to disclose its site visit to the general public. Under mandatory disclosure, an institution must publicly disclose its site visit as implemented by the SZSE. We demonstrate that messages take a binary form under this application.

Trading and Pricing. Let $\tilde{p}$ denote the risky asset's date-1 price in the financial market. Conditional on the acquired signal $\tilde{y}_j$ and observed messages $\{m_1, ..., m_J\}$, investor j places market order $\tilde{x}_j$ to maximize her expected trading profits $E\left[\tilde{x}_j\left(\tilde{v}-\tilde{p}\right)|\tilde{y}_j; m_1, ..., m_J\right]$. As is typical, noise traders place their random order $\tilde{u}$, where $\tilde{u} \sim N(0, \sigma_u^2)$ (with $\sigma_u > 0$) and $\tilde{u}$ is independent of all other random shocks. Thus, $\tilde{\omega} = \sum_{j=1}^{J} \tilde{x}_j + \tilde{u}$ is the total order flows observed by the market maker who sets the asset's price $\tilde{p}$ according to the weak-efficiency rule:

$$\tilde{p} = E[\tilde{v}|\tilde{\omega}; m_1, ..., m_J]. \tag{1}$$

Equilibrium Concept. At date-1 trading, we consider a linear equilibrium in which both investors' trading rules and the market maker's pricing rule are linear functions and the messages affect the coefficients in the trading and pricing rules. That is, investor j's trading rule takes the form $\alpha_j(m_1, ..., m_J) \cdot \tilde{y}_j$, and the market maker's pricing rule takes the form $\lambda(m_1, ..., m_J) \cdot \tilde{\omega}$, which essentially extends the standard linear equilibrium in Kyle's (1985) trading game by noting that market agents' beliefs about investors' information precision levels are affected by the messages. When computing the trading equilibrium, we must specify a belief function $\hat{h}_j(m_j)$, which stipulates how the agents use message m_j to update their belief $\hat{h}_j$ about investor j's information precision h_j. In most equilibrium concepts of game theory (e.g., perfect Bayesian equilibrium), belief functions are part of the equilibrium definition and pinned down by on-equilibrium belief consistency and off-equilibrium belief specification. The characterization of these belief functions, thus, requires knowledge of the communication game's details.

At date-0's information-acquisition stage, each investor chooses her level of information precision to maximize her unconditional expected profits evaluated under date-1 trading's equilibrium. In making this information-acquisition decision, the investor takes her rivals' information choices as given but understands that her information choice will be communicated to the public through the message function $m_j(h_j)$ and belief function $\hat{h}_j(m_j)$, which can potentially influence her rivals' trading rules and the market maker's pricing rule at a later trading stage. Since this section treats the communication technology at an abstract level without specifying the communication game, we cannot characterize how the message and belief functions are exactly determined. Therefore, we provide a formal equilibrium definition for exogenous message and belief functions. We also consider pure strategies of information acquisition since allowing mixed strategies of information acquisition in settings with unobservable information acquisition breaks down the linearity of agents' inference problems in the trading game, rendering the model no longer analytically tractable.

Definition 1 (Equilibrium with exogenous message functions and belief functions)**.** *An equilibrium with exogenous message functions $\{m_j(h_j)\}_{j=1}^J$ and belief functions $\{\hat{h}_j(m_j)\}_{j=1}^J$ consists of the investors' date-0 information precision levels $\left\{h_j^*\right\}_{j=1}^J$, the investors' date-1 trading rules $\{\alpha_j\left(m_1, ..., m_J\right) \cdot \tilde{y}_j\}_{j=1}^J$, and the market maker's date-1 pricing rule $\lambda\left(m_1, ..., m_J\right) \cdot \tilde{\omega}$, such that:*

(1) *On date 0, information-precision levels $\left\{h_j^*\right\}_{j=1}^J$ form a Nash equilibrium, i.e.,*

$$h_j^* = \arg\max_{h_j} E\left[\alpha_j \tilde{y}_j \left(\tilde{v} - \lambda \alpha_j \tilde{y}_j - \lambda \sum_{k \neq j}^J \alpha_k \tilde{y}_k\right) - C(h_j)\right], \forall j, \tag{2}$$

where $\alpha_j \equiv \alpha_j\left(m_1\left(h_1^*\right), ..., m_j\left(h_j\right), ..., m_J\left(h_J^*\right)\right)$, $\lambda \equiv \lambda\left(m_1\left(h_1^*\right), ..., m_j\left(h_j\right), ..., m_J\left(h_J^*\right)\right)$, $\tilde{y}_j = \tilde{v} + \tilde{e}_j$ with $\tilde{e}_j \sim N\left(0, h_j^{-1}\right)$, $\tilde{y}_k = \tilde{v} + \tilde{e}_k$ with $\tilde{e}_k \sim N\left(0, h_k^{*-1}\right)$ (for $k \neq j$), and $\{\tilde{v}, \tilde{e}_1, ..., \tilde{e}_J\}$ are mutually independent.

(2) *On date 1, each investor's trading rule maximizes her expected profits conditional on her pri-*

vate information $\tilde{y}_j$ and messages $\{m_1, ..., m_J\}$, given other investors' trading rules, the market maker's pricing rule, and the messages, i.e.,

$$\alpha_j \tilde{y}_j = \arg\max_{x_j} E\left[x_j \left(\tilde{v} - \lambda x_j - \lambda \sum_{k \neq j}^{J} \alpha_k \tilde{y}_k \right) \middle| \tilde{y}_j; m_1, ..., m_J \right], \forall j; \tag{3}$$

and the marker maker's pricing rule is set according to (1) given all investors' trading rules, i.e.,

$$\lambda = \frac{Cov\left(\tilde{v}, \sum_{j=1}^{J} \alpha_j \tilde{y}_j \middle| m_1, ..., m_J \right)}{Var\left(\tilde{v}, \sum_{j=1}^{J} \alpha_j \tilde{y}_j \middle| m_1, ..., m_J \right)}. \tag{4}$$

In computing the conditional moments in equations (3) and (4), agents use the belief function $\hat{h}_j(m_j)$ to update their belief about investor j's information precision.

2 Secret Versus Overt Information Acquisition

In this section, we analyze and compare two benchmark economies: One in which all investors engage in secret information acquisition and the second in which all engage in overt information acquisition. The analysis in this section introduces the details surrounding equilibrium derivations and transparently illustrates how secret information acquisition differs from overt information acquisition. It also helps build clean intuitions when exploring the disclosure of corporate site visits in Section 3.

As mentioned in Section 1, under the secret benchmark economy, the message m_j always takes value $\varnothing$, independent of investor j's actual information precision h_j; that is, the message function is $m_j(h_j) = \varnothing$. The message $\varnothing$ must be on equilibrium and, so, the other agents' beliefs $\hat{h}_j$ about h_j are pinned down by the on-equilibrium belief consistency: $\hat{h}_j(\varnothing) = h_j^*$, where h_j^* is the equilibrium value of h_j. Under the overt benchmark economy, m_j perfectly reveals h_j. Thus, the message function is $m_j(h_j) = h_j$ and the belief function is accordingly $\hat{h}_j(m_j) = m_j$. In the language of game theory, the information sets under overt acquisition degenerate to a singleton and, thus, the belief distributions assign probability one to the single node.

2.1 Monopolistic Investor $(J = 1)$

In this monopolistic setting, only the interaction between the investor and market maker prevails. Analyzing this setting allows for a stark, intuitive comparison of secret versus overt information acquisition.

2.1.1 Solving the Model

Since only one investor exists, we drop the subscript j and use $h, \tilde{y}$, and $\tilde{x}$ to denote the investor's information precision, private signal, and order flow, respectively. The investor's linear trading rule is given by $\tilde{x} = \alpha\tilde{y}$, and the market maker's linear pricing rule is given by $\tilde{p} = \lambda\tilde{\omega}$. The equilibrium is characterized by the tuple (h, λ, α). We use $(h_s, \alpha_s, \lambda_s)$ and $(h_o, \alpha_o, \lambda_o)$ to denote equilibrium variables under secret and overt information acquisition, respectively.

On date 1, the investor takes the market maker's linear pricing rule as given and computes her conditional expected trading profits as $E\left[\tilde{x}\left(\tilde{v} - \tilde{p}\right)|\tilde{y}; m\right] = -\lambda\tilde{x}^2 + \frac{h\tilde{y}}{1+h}\tilde{x}$. As in Kyle (1985, p.1319), this quadratic objective, implied by the linear pricing rule, rules out mixed trading strategies and yields a unique linear trading rule given by $\tilde{x} = \alpha\tilde{y}$,[4] with

$$\alpha = \frac{h}{2(1+h)\lambda}. \tag{5}$$

The market maker forms belief $\hat{h}$ about the investor's precision h according to the message m. As explained above, $\hat{h} = h_s$ under secret information acquisition and $\hat{h} = h$ under overt information acquisition.[5] In both cases, the market maker takes belief $\hat{h}$ to form conjecture $\hat{\alpha}$ about the investor's optimal trading strategy according to (5): $\hat{\alpha} = \frac{\hat{h}}{2(1+\hat{h})\lambda}$. Equipped with the conjectured $\hat{h}$ and $\hat{\alpha}$, the market maker then chooses λ according to pricing rule (1), yielding $\lambda = \frac{\hat{\alpha}}{\hat{\alpha}^2\left(1+1/\hat{h}\right)+\sigma_u^2}$. Combining this equation with $\hat{\alpha} = \frac{\hat{h}}{2(1+\hat{h})\lambda}$, we compute the market maker's pricing rule as a

[4] Confirming that the second-order condition (SOC) is negative can be straightforward so long as $\lambda > 0$. Thus, this trading strategy, indeed, maximizes the investor's profits. In the remainder of the paper, the SOC is always checked.

[5] Under secret information acquisition, using message function $m(h) = \varnothing$ and belief function $\hat{h}(\varnothing) = h_s$, we have $\hat{h}(m(h)) = \hat{h}(\varnothing) = h_s$ for any actual level h of investor's information precision. Similarly, under overt information acquisition, we have $\hat{h}(m(h)) = m(h) = h$, $\forall h$, by $m(h) = h$ and $\hat{h}(m) = m$.

function of her belief regarding the investor's precision, which generates the following:

$$\lambda = \frac{\sqrt{\hat{h}}}{2\sigma_u\sqrt{1+\hat{h}}}. \tag{6}$$

Next, we return to date 0 to determine the investor's optimal information-acquisition decision. We compute the investor's unconditional expected profits, which result from her optimal trading rule as $\lambda E(\tilde{x}^2) = \lambda\alpha^2(1+1/h)$. Using α in (5) and subtracting the information-acquisition cost, we obtain the investor's expected net trading profits π as a function of (h, λ):

$$\pi(h,\lambda) = \frac{h}{4\lambda(1+h)} - C(h). \tag{7}$$

Inserting the linear cost function $C(h) = c \cdot h$ and using equation (6) to replace λ in the above equation, we can reexpress π as a function of $(h, \hat{h})$ as follows:

$$\pi(h,\hat{h}) = \frac{h\sigma_u\sqrt{1+\hat{h}}}{2(1+h)\sqrt{\hat{h}}} - c \cdot h. \tag{8}$$

Under secret information acquisition, $\hat{h}$ is fixed at the equilibrium precision h_s. When maximizing the investor's profits, we fix $\hat{h}$ in (8) and take the *partial derivative* with respect to h. Setting this partial derivative at 0 yields the investor's optimal information precision: $h_s = \sqrt{\frac{\sigma_u}{2c}}\left(1+\frac{1}{\hat{h}}\right)^{1/4} - 1$. Finally, replacing $\hat{h} = h_s$ completes the equilibrium's characterization. In contrast, under overt information acquisition, $\hat{h} = h$. Now, the pricing rule (6) adjusts with h, which the investor anticipates; in turn, her information-acquisition incentive is changed. Specifically, we first replace $\hat{h}$ with h in equation (8) and then take the *total derivative* with respect to h to maximize the investor's profits. Taken together, the key difference between the secret and overt information-acquisition incentives boils down to whether or not the investor's precision choice can influence the pricing rule λ by influencing the market maker's belief $\hat{h}$.

The following lemma summarizes the equilibrium in the monopolistic investor setting. Throughout our analysis, we impose the linear cost function $C(h) = c \cdot h$, and all formal results are stated under this assumption.

Lemma 1. *Suppose $J = 1$. Then:*

(1) *If information is acquired secretly, there exists a unique equilibrium. The equilibrium information*

precision, h_s, is the unique solution to the following equation:

$$h_s(1+h_s)^3 = \frac{\sigma_u^2}{4c^2}. \tag{9}$$

The investor's trading strategy is $\tilde{x} = \alpha_s \tilde{y}$ with $\alpha_s = \sigma_u \sqrt{\frac{h_s}{1+h_s}}$, and the pricing rule is $\tilde{p} = \lambda_s \tilde{\omega}$ with $\lambda_s = \frac{1}{2\sigma_u}\sqrt{\frac{h_s}{1+h_s}}$.

(2) *If information acquisition is acquired overtly, there exists a unique equilibrium. The equilibrium information precision, h_o, is the unique solution to the following equation:*

$$h_o(1+h_o)^3 = \frac{\sigma_u^2}{16c^2}. \tag{10}$$

The investor's trading strategy is $\tilde{x} = \alpha_o \tilde{y}$ with $\alpha_o = \sigma_u \sqrt{\frac{h_o}{1+h_o}}$, and the pricing rule is $\tilde{p} = \lambda_o \tilde{\omega}$ with $\lambda_o = \frac{1}{2\sigma_u}\sqrt{\frac{h_o}{1+h_o}}$.

2.1.2 Equilibrium Comparison

Information Precision. We can show that an overt monopolist investor always acquires less information than a secret monopolist investor does. The key difference between overt and secret information acquisition is that the overt investor considers and remains sensitive to the market maker's response to her information acquisition. To illustrate this point, we apply the chain rule to $\pi(h, \lambda)$ in equation (7) and express the investor's information-acquisition incentive as follows:

$$\frac{d\pi}{dh} = \underbrace{\frac{\partial \pi}{\partial h}}_{\text{standard trade-off (?)}} + \underbrace{\frac{\partial \pi}{\partial \lambda}\frac{\partial \lambda}{\partial h}}_{\substack{\text{overt: pricing effect } (<0) \\ \text{secret: } (=0)}}, \tag{11}$$

where $\frac{\partial \pi}{\partial h} = \frac{1}{4\lambda(1+h)^2} - c$ and $\frac{\partial \pi}{\partial \lambda} = -\frac{h}{4\lambda^2(1+h)} < 0$. By equation (6), we have $\frac{\partial \lambda}{\partial h} = \frac{1}{4\sigma_u(1+h)\sqrt{h(1+h)}} > 0$ for an overt investor ($\hat{h} = h$) and $\frac{\partial \lambda}{\partial h} = 0$ for a secret investor ($\hat{h} = h_s$).

The first term in (11) captures the standard trade-off faced by an investor considering the acquisition of information: Other things being equal (i.e., λ being fixed), more precise information helps the investor better forecast the risky asset's fundamental value $\tilde{v}$, but this acquisition costs more resources. This trade-off is present under both secret and overt information acquisition.

The second term in (11) is an additional effect that only operates in the case of overt information acquisition, coming specifically from the market maker's strategic response to the investor's information acquisition. We label this term the *pricing effect*. Specifically, as the overt investor acquires more precise information and trades more aggressively, the market maker knows that she will be more exposed to private information when providing liquidity; hence, her adverse-selection concern worsens. In response, the market maker sets a higher λ (i.e., $\frac{\partial \lambda}{\partial h} > 0$). Facing this steepened pricing schedule, the overt investor trades less aggressively and makes a lower profit (i.e., $\frac{\partial \pi}{\partial \lambda} < 0$). The strategic response of the market maker, therefore, reduces the overt investor's incentive to acquire information (i.e., $\frac{\partial \pi}{\partial \lambda}\frac{\partial \lambda}{\partial h} < 0$).[6] Under secret information acquisition, this strategic effect is muted since the market maker's pricing rule λ is not affected by the investor's actual information acquisition (i.e., $\frac{\partial \lambda}{\partial h} = 0$ when $\hat{h}$ is fixed at h_s). Taken together, the second term in (11) implies that, in equilibrium, an overt monopolistic investor acquires less information than a secret monopolistic investor.

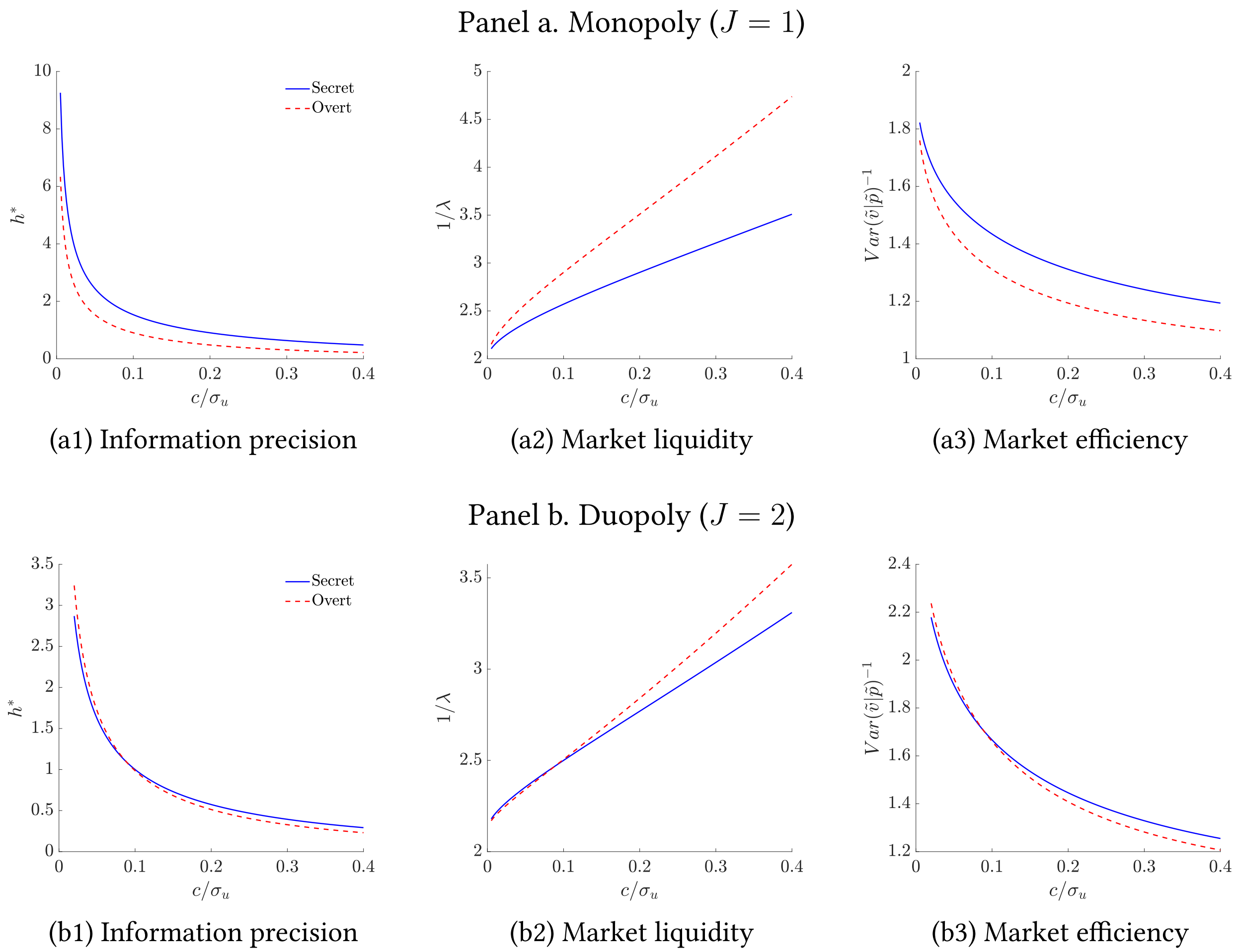
Market Quality. We now examine two market-quality variables: Market liquidity and market efficiency. These two variables are central to regulatory debates and have been extensively studied in the literature (e.g., Easley, O'Hara, and Yang, 2016). We measure market liquidity using Kyle's λ: A more liquid market is associated with a smaller λ. Intuitively, λ captures the price impact of order flows and is, thus, negatively related to market depth. Note that, in equilibrium, market liquidity $1/\lambda$ is inversely related to equilibrium information precision through equation (6) under both overt ($\hat{h} = h_o$) and secret ($\hat{h} = h_s$) information acquisition. Since $h_o < h_s$, $1/\lambda_o > 1/\lambda_s$; that is, the overt market is more liquid than the secret market.

Market efficiency captures how much information is incorporated into the asset price. We measure market efficiency by the posterior precision of the asset payoff conditional on its price: $Var(\tilde{v}|\tilde{p})^{-1}$. Intuitively, when the price aggregates a great deal of information, the residual uncertainty of the fundamental $\tilde{v}$, conditional on the price $\tilde{p}$, is low; thus, market efficiency is high. Given h, market efficiency is determined as $Var(\tilde{v}|\tilde{p})^{-1} = \frac{2(h+1)}{h+2}$, which increases in h. As $h_o < h_s$, it immediately follows that $Var(\tilde{v}|\tilde{p})_o^{-1} < Var(\tilde{v}|\tilde{p})_s^{-1}$. Intuitively, since the overt in-

[6] This result can be reversed in a scenario with multiple investors, which we revisit in the next subsection (see Lemma 3).

vestor acquires less information than the secret investor, the price in the overt market, of course, contains less information about the asset's fundamental value. Proposition 1 and Panel (a) of Figure 2 summarize the above discussions:

Proposition 1 (Secret versus overt information acquisition in the monopoly)**.** *Suppose* $J = 1$*. Relative to secret information acquisition, overt information acquisition leads to less precise information, higher market liquidity, and lower market efficiency. That is,* $h_o < h_s, 1/\lambda_o > 1/\lambda_s$*, and* $Var(\tilde{v}|\tilde{p})_o^{-1} < Var(\tilde{v}|\tilde{p})_s^{-1}$.



This figure plots investors' equilibrium information acquisition, market liquidity, and market efficiency against c/σ_u under monopoly (Panel a) and duopoly (Panel b). The solid and dashed lines depict the benchmarks with secret and overt information acquisition, respectively.

Figure 2: Secret versus overt information acquisition

2.2 Oligopolistic Investors ($J \geq 2$)

In this section, we study a setting that features oligopolistic investors ($J \geq 2$). Here, we introduce interactions among strategic investors and investigate their interplay with the pricing effect. The following lemma extends Lemma 1 to the general case with $J \geq 2$. We focus on symmetric equilibria in which investors end up with the same information precision on date 0 and the same trading rule on date 1. The computation parallels that of Section 2.1.1 and is, thus, relegated to the appendix.

Lemma 2. (1) *If information is acquired secretly, there exists a unique symmetric equilibrium. The equilibrium information precision h_s is the unique solution to*

$$h_s(1+h_s)\left(2+h_s+h_sJ\right)^2 = \frac{\sigma_u^2}{Jc^2}. \tag{12}$$

The investor's trading strategy is $\tilde{x}_j = \alpha_s \tilde{y}_j$ with $\alpha_s = \sigma_u \sqrt{\frac{h_s}{(1+h_s)J}}$, and the pricing rule is $\tilde{p} = \lambda_s \tilde{\omega}$ with $\lambda_s = \frac{\sqrt{Jh_s(1+h_s)}}{\sigma_u(2+(J+1)h_s)}$.

(2) *If information is acquired overtly, there exists a unique symmetric equilibrium. The equilibrium information precision h_o is the unique solution to*

$$\frac{h_o(1+h_o)(2+h_o)^2(2+h_o+h_oJ)^4}{\left((6J^2-J-3)h_o^2+(4J^2+10J-8)h_o+8J-4\right)^2} = \frac{\sigma_u^2}{4J^3c^2}. \tag{13}$$

The investor's trading strategy is $\tilde{x}_j = \alpha_o \tilde{y}$ with $\alpha_o = \sigma_u \sqrt{\frac{h_o}{(1+h_o)J}}$, and the pricing rule is $\tilde{p} = \lambda_o \tilde{\omega}$ with $\lambda_o = \frac{\sqrt{Jh_o(1+h_o)}}{\sigma_u(2+(J+1)h_o)}$.

2.2.1 Pricing Effect versus Competition Effect

To understand the consequences of secret versus overt information acquisition, let us examine a representative investor j's information-acquisition incentive. Let λ and α_{-j} denote the price impact set by the market maker and the trading intensity of other investors except for j, respectively. We can compute investor j's expected date-0 profit evaluated at the date-1 trading equilibrium as follows:

$$\pi_j(h_j,\lambda,\alpha_{-j}) = \frac{h_j\left[1-(J-1)\,\alpha_{-j}\lambda\right]^2}{4\lambda\,(1+h_j)} - c\cdot h_j, \tag{14}$$

which extends equation (7) to the oligopolistic setting. The difference between secret and overt information acquisition lies in whether investor j's information choice affects the market maker's pricing rule λ and other investors' trading rule α_{-j} by influencing these agents' beliefs $\hat{h}_j$ about investor j's information precision h_j. Again, under secret information acquisition, $\hat{h}_j$ is fixed at the equilibrium level, i.e., $\hat{h}_j = h_s$. Under overt information acquisition, the market maker and other investors can perfectly observe h_j and, hence, $\hat{h}_j = h_j$.

We, next, follow similar steps as those discussed in Section 2.1.1 to derive the expressions of λ and α_{-j} as functions of $\hat{h}_j$. The idea is to compute the trading equilibrium, given the market's belief $\hat{h}_j$. Specifically, given the market maker's pricing rule λ and the other investors' trading rule α_{-j}, investor j's optimal trading rule is $\alpha_j = \frac{h_j(1-(J-1)\alpha_{-j}\lambda)}{2(1+h_j)\lambda}$. Hence, given belief $\hat{h}_j$, other agents believe that investor j's optimal trading rule is $\hat{\alpha}_j = \frac{\hat{h}_j(1-(J-1)\alpha_{-j}\lambda)}{2(1+\hat{h}_j)\lambda}$. Taking a generic non-$j$ investor – investor k (where $k \neq j$) – we solve for her optimal trading rule given $\hat{\alpha}_j$, λ, and α_{-j}, and then impose symmetry ($\alpha_k = \alpha_{-j}$) to obtain $\alpha_{-j} = \frac{h^*(1-\hat{\alpha}_j\lambda)}{(2+h^*J)\lambda}$, where h^* denotes the equilibrium precision level chosen by the other $J-1$ investors (i.e., $h^* = h_s$ under secret information acquisition, and $h^* = h_o$ under overt information acquisition). Given $\hat{h}_j$, h^*, $\hat{\alpha}_j$, and α_{-j}, we can compute the market maker's pricing rule as $\lambda = \frac{(J-1)\alpha_{-j}+\hat{\alpha}_j}{((J-1)\alpha_{-j}+\hat{\alpha}_j)^2+(J-1)\alpha_{-j}^2/h^*+\hat{\alpha}_j^2/\hat{h}_j+\sigma_u^2}$. Combining this equation with $\hat{\alpha}_j = \frac{\hat{h}_j(1-(J-1)\alpha_{-j}\lambda)}{2(1+\hat{h}_j)\lambda}$ and $\alpha_{-j} = \frac{h^*(1-\hat{\alpha}_j\lambda)}{(2+h^*J)\lambda}$, we compute λ and α_{-j} as functions of $\hat{h}_j$ as follows:

$$\lambda = \frac{\sqrt{\hat{h}_j^2(4+h^{*2}J+h^*(J+3))+\hat{h}_j(4+4h^*J+h^{*2}(4J-3))+4h^*(1+h^*)(J-1)}}{\left(4+2h^*J+\hat{h}_j(4+h^*+h^*J)\right)\sigma_u}, \tag{15}$$

$$\alpha_{-j} = \frac{h^*\left(\hat{h}_j+2\right)\sigma_u}{\sqrt{\hat{h}_j^2(4+h^{*2}J+h^*(J+3))+\hat{h}_j(4+4h^*J+h^{*2}(4J-3))+4h^*(1+h^*)(J-1)}}. \tag{16}$$

Note that equation (15) extends equation (6) to the oligopolistic setting.

Equipped with the expressions of λ and α_{-j} in (15) and (16), we can apply the chain rule to

$\pi_j(h_j, \lambda, \alpha_{-j})$ in (14) and decompose investor j's information-acquisition incentive as follows:

$$\frac{d\pi_j}{dh_j} = \underbrace{\frac{\partial \pi_j}{\partial h_j}}_{\text{standard trade-off (?)}} + \underbrace{\underset{<0}{\frac{\partial \pi_j}{\partial \lambda}} \underset{?}{\frac{\partial \lambda}{\partial h_j}}}_{\substack{\text{overt: pricing effect (?)} \\ \text{secret: (=0)}}} + \underbrace{\underset{<0}{\frac{\partial \pi_j}{\partial \alpha_{-j}}} \underset{<0}{\frac{\partial \alpha_{-j}}{\partial h_j}}}_{\substack{\text{overt: competition effect (>0)} \\ \text{secret: (=0)}}}, \tag{17}$$

which extends equation (11) in the monopolistic setting to the oligopolistic setting. As in (11), the first term in (17) captures the standard trade-off that exists in acquiring information: Other things being equal (i.e., λ and α_{-j} being fixed), acquiring more precise information helps investor j predict the asset's fundamental value $\tilde{v}$ and the rival investors' demands but, simultaneously, she incurs a higher cost. The second term in (17) is the pricing effect resulting from the market maker's strategic response, as discussed in (11) for the monopolistic setting (only, here, the positivity or negativity of the pricing effect is ambiguous while it is always negative when $J = 1$). Finally, the third term in (17) represents the strategic response of other investors. We label this effect the *competition effect*, an additional effect present in the oligopolistic setting.

The first effect is present under both secret and overt information acquisition while the second and third effects only prevail under overt information acquisition. As a result, whether overt information acquisition generates more precise information in equilibrium than secret information acquisition depends on the interaction of the pricing and competition effects. We next sign these two effects in the overt economy.

We can show that the competition effect is always positive (i.e., $\frac{\partial \pi_j}{\partial \alpha_{-j}} \frac{\partial \alpha_{-j}}{\partial h_j} > 0$). First, investor j's profits decrease as her rivals trade more aggressively (i.e., $\frac{\partial \pi_j}{\partial \alpha_{-j}} < 0$). Second, as investor j acquires more precise information, her rivals trade less aggressively on their own private information (i.e., $\frac{\partial \alpha_{-j}}{\partial h_j} < 0$) because they expect investor j to trade more aggressively according to her more precise information. Overall, we have $\frac{\partial \pi_j}{\partial \alpha_{-j}} \frac{\partial \alpha_{-j}}{\partial h_j} > 0$, so that the competition effect tends to strengthen an overt investor's information-acquisition incentive.

However, the pricing effect is ambiguous in the overt oligopolistic setting: $\frac{\partial \pi_j}{\partial \lambda} \frac{\partial \lambda}{\partial h_j}$ can be either positive or negative. We can show that $\frac{\partial \pi_j}{\partial \lambda} < 0$; that is, a steeper pricing schedule lowers investor j's profits. However, the sign of $\frac{\partial \lambda}{\partial h_j}$ can be either positive or negative, depending on the number of investors and their information quality.

To illustrate ideas, we use equation (15) to investigate the positivity or negativity of $\frac{\partial \lambda}{\partial h_j}$ when the information precision is the same across all investors. Recall that λ is set by the market maker as the regression coefficient of $\tilde{v}$ on the total order flow $\tilde{\omega}$, given her knowledge about investors' information precision levels in the overt economy. This regression coefficient is the product of two terms: The correlation between $\tilde{\omega}$ and $\tilde{v}$ and the ratio of the volatility of $\tilde{v}$ to the volatility of $\tilde{\omega}$. An increase in h_j affects these two terms in opposite ways. First, the total order flow contains more fundamental information and, thus, becomes more correlated with the fundamental value, leading to a higher λ. Second, since the total order flow becomes more sensitive to $\tilde{v}$, its volatility becomes higher, which lowers λ. When there are many rivals and their information quality is high, investors share largely common information and compete aggressively. In this case, this aggressive trading already implies a high correlation between $\tilde{\omega}$ and $\tilde{v}$ and, thus, the correlation force due to the increased h_j is weak, implying that $\frac{\partial \lambda}{\partial h_j} < 0$ (and, hence, $\frac{\partial \pi_j}{\partial \lambda}\frac{\partial \lambda}{\partial h_j} > 0$, a positive pricing effect). By contrast, when there are only a few rivals, or when their information is coarse, investors trade conservatively and the correlation force dominates, which implies that $\frac{\partial \lambda}{\partial h_j} > 0$ (and hence $\frac{\partial \pi_j}{\partial \lambda}\frac{\partial \lambda}{\partial h_j} < 0$, a negative pricing effect).

Note that, in the monopolistic setting, the above-two forces are also active, but only the monopolistic investor's conservative trading renders the correlation force dominant. We can connect the oligopolistic setting to the monopolistic setting in the following way: When the number J of investors is small, each oligopolistic investor behaves naturally like a monopolistic investor and, so, the pricing effect is negative as in Section 2.1.2. When J is large but h^* is low, each investor has some degree of monopoly power because part of her information is known only to her; again, each investor behaves like a local monopolist of her own private signal, and the pricing effect is also negative.

Lemma 3. *In the overt oligopolistic setting, the following properties hold:*

(1) *The competition effect is always positive. That is,* $\frac{\partial \pi_j}{\partial \alpha_{-j}}\frac{\partial \alpha_{-j}}{\partial h_j} > 0$.

(2) *The pricing effect is ambiguous.*

- *For* $J \leq 3$, $\frac{\partial \pi_j}{\partial \lambda}\frac{\partial \lambda}{\partial h_j}\Big|_{h_j = h^*} < 0$.
- *For* $J > 3$, $\frac{\partial \pi_j}{\partial \lambda}\frac{\partial \lambda}{\partial h_j}\Big|_{h_j = h^*} < 0$ *if and only if* $h^* < \frac{2}{J-3}$.

2.2.2 Secret versus Overt Information Acquisition

Lemma 3 suggests that, when J is large and h^* is high, the pricing and competition effects work in the same direction; in equilibrium, overt information acquisition leads to more precise information than secret information acquisition does. We can generalize this result and show that, for any value of J, the equilibrium precision level of overt information is higher than that of secret information if c/σ_u is lower than a threshold value (so that the equilibrium information precision h^* is high). Panel (b1) of Figure 2 graphically illustrates this result for $J = 2$. In this duopoly setting, the pricing effect works against the competition effect (see Lemma 3). Nonetheless, when c/σ_u is low, investors choose to acquire precise information in equilibrium so that the competition among them intensifies, implying that the competition effect is particularly strong and dominates the pricing effect.

Given precision level h, we can compute market efficiency in equilibrium as $Var(\tilde{v}|\tilde{p})^{-1} = \frac{2+h+hJ}{2+h}$. Intuitively, market efficiency increases with h, and, as a result, comparisons of market efficiency mirror comparisons of information precision between secret and overt acquisition.

Market liquidity is still inversely measured by Kyle's λ. According to Lemma 2, under secret and overt information acquisition, λ is connected to equilibrium information precision h^* as follows: $\lambda = \frac{\sqrt{h^*(1+h^*)J}}{\sigma_u(2+(J+1)h^*)}$. The way that h^* affects λ shares some similarities with Lemma 3, in which we vary one investor's information precision h_j and examine how the market maker adjusts λ. In the present case, we vary the information precision of all investors, but a similar result prevails: When $J \leq 3$, λ increases in h^* (i.e., $\frac{\partial \lambda}{\partial h^*} > 0$) whereas when $J > 3$, λ is hump-shaped in h^*; that is, $\frac{\partial \lambda}{\partial h^*} > 0$ if and only if $h^* < \frac{2}{J-3}$. If we focus on the case with $J \leq 3$, then comparisons of λ immediately follow comparisons of h^* between secret and overt information acquisition.

Panel (b) of Figure 2 and the following proposition summarize our results.

Proposition 2 (Secret versus overt information acquisition in oligopoly). *Define* $\zeta \equiv \frac{1}{(2+\zeta_0+\zeta_0 J)\sqrt{\zeta_0(1+\zeta_0)J}}$ *where* $\zeta_0 \equiv \frac{4-J+\sqrt{17J^2-20J+4}}{4J^2-3J-3}$. *Then:*

(1) *Overt investors acquire more precise information than secret investors if and only if* $c/\sigma_u < \zeta$*; that is,* $h_o > h_s$ *if and only if* $c/\sigma_u < \zeta$.

(2) *An overt market is more informationally efficient than a secret market if and only if* $c/\sigma_u < \zeta$*; that is,* $Var(\tilde{v}|\tilde{p})_o^{-1} > Var(\tilde{v}|\tilde{p})_s^{-1}$ *if and only if* $c/\sigma_u < \zeta$.

(3) *Suppose that* $J \leq 3$. *An overt market is less liquid than a secret market if and only if* $c/\sigma_u < \zeta$*; that is,* $1/\lambda_o < 1/\lambda_s$ *if and only if* $c/\sigma_u < \zeta$.

3 Site Visits: Voluntary and Mandatory Disclosures of Information Acquisition

In this section, we apply our model to study one significant setting that fosters information acquisition: Corporate site visits by financial institutions. We first discuss the institutional details of corporate site visits and demonstrate how our model connects to this concrete setting. Then, we explore the voluntary and mandatory disclosures of site visits.

3.1 Institutional Background of Site Visits

"Site visits refer to investors' field trips to corporate headquarters and production facilities. During these trips, investors observe firms' operations and talk to employees. Corporate site visits are an increasingly important type of information-acquisition activity for institutional investors... The 2012 Institutional Investor's All-Europe Research Team survey shows that institutional investors rank corporate site visits to be more important than one-on-one meetings with the management and analysts' research reports in terms of acquiring information about firms" (Cheng et al., 2019, p.360). As emphasized by Chen et al. (2022), face-to-face interactions with firms allow investors to collect valuable soft information regarding their corporate culture, employee morale, firm strategies, and more. This translates into valuable information that comes to influence agents' investment decisions.[7]

The observability of site visits can be affected by voluntary and mandatory disclosures. Financial institutions often voluntarily disclose their corporate site visits. For instance, in February 2020, UK investment company Castlefield disclosed on its website the site visit it conducted with Alumasc, another UK-based supplier of premium building products.[8] The fund emphasized that, though costly, such visits helped it learn more about the company: "Taking the time to attend these visits often allows us to gain more insight into the processes and products involved in its operations and the potential to speak to other senior managers that do not typically meet with investors." The fund's resulting report, which can be viewed as hard evidence of this specific site visit, was posted on a public domain and written with serious intent. In this sense, the fund's disclosure is credible and verifiable.

Given the potential for such transparency across the board, there has been an ongoing regulatory debate on whether all corporate site visits should be made public. The idea can be viewed as an extension of Reg FD, which prevents the selective disclosure of material non-public information to level the playing field. For instance, Solomon and Soltes (2015) propose that firms should publish site visit information so that all market participants are aware of the visits.[9] Although the U.S. has not implemented this proposal, China has. In 2006, the SZSE issued its own form of Reg FD, stating that SZSE-listed firms should not disclose material non-public information only to select investors. In the same year, the SZSE began requiring the same listed firms to disclose

[7] Using a unique set of proprietary private-meeting records of one firm traded on the New York Stock Exchange, Solomon and Soltes (2015) show that private meetings help hedge funds make more informed trading decisions. Cheng et al. (2019) and Chen et al. (2022) find similar results for mutual funds that conduct site visits with Chinese companies: "Mutual fund company visits affect investment decisions because such visits provide valuable information to mutual funds" (Chen et al., 2022, p.4).

[8] See https://www.castlefield.com/news-media/blog/site-visit-to-timloc-a-division-of-the-stock-alumasc/. Cheng et al. (2013, p.9) also discuss studies exploring voluntarily disclosed site visits in the U.S. capital market.

[9] See, for instance, Ng and Troianovski (2015), Bengtzen (2017), Soltes (2018), and Bowen et al. (2020), for more discussions on the debate.

site visits in their annual reports. In 2012, the SZSE took further steps to demand all listed firms to disclose site visits within two trading days of the events through a designed web portal called Hu Dong Yi.[10] Both the two-day requirement and accessible web portal make disclosures near real-time. The disclosure itself can be brief, typically consisting of no more than the visit's date, location, outside participants' names and affiliations, corporate employees' names and titles, and a short summary of questions and answers discussed during the site visit.[11]

3.2 Mapping Site Visits to the Model

Investors acquire fundamental information about a company through a variety of methods, such as studying financial reports, attending conference calls, and conducting site visits. Each method can approximately be represented by a segment of the cost function curve, $C(h_j)$. The left end of the segment shows that, at a fixed cost, the method delivers a minimum level of information precision. An investor can expend more resources to increase her precision level but only to a certain limit, which is represented by the right end of the segment. In Section S1 of the online appendix, we provide more details on this "segment" representation of information acquisition.

For site visits, the investor physically travels to a firm and observes its operations, which then allows her to collect a minimum precision level of information, denoted by constant $\bar{h}$. The visit details determine the actual value of the investor's precision h_j, which, in turn, increases with the investor's efforts during the visit. If the investor is more attentive in conversations, visits more facilities, and/or engages more company employees, the information collected becomes more accurate. Since visit details are difficult to credibly convey to the public, disclosing site visits, whether voluntarily or mandatorily, primarily discloses the fact that the investor has conducted a visit. Accordingly, a message disclosing investor j's site visit, denoted by $m_j = SV$, effectively

[10] The web portal is http://irm.cninfo.com.cn/szse/index_en.html.

[11] Examples of disclosure notes can be found in Bowen et al. (2018), Cheng et al. (2019), and Chen et al. (2022). The disclosure also classifies meetings into various categories, such as site visits, in-house investor meetings, analyst day meetings, online interactions, roadshows, and remote conference calls. As pointed out by Bowen et al. (2018) and Bowen et al. (2020), firms use "site visits" and "in-house investor meetings" interchangeably to describe their activities; so, we use "site visits" to refer to both categories. These two categories of activities account for approximately 90% of all public-relation activities reported on the SZSE website.

reveals that investor j's precision h_j is bounded below by $\bar{h}$ (i.e., $h_j \geq \bar{h}$).[12]

Under voluntary disclosure, if investor j conducts a site visit, she can decide whether or not to disclose it. As discussed above, when investor j discloses (i.e., $m_j = SV$), other agents understand that $h_j \geq \bar{h}$ and will accordingly use this knowledge to form their beliefs about h_j. When investor j does not disclose (denoted by $m_j = \varnothing$), other agents do not directly observe any information about h_j (i.e., $h_j \geq 0$). In the voluntary disclosure setting, if the investor does not conduct a visit, she is incapable of credibly conveying this fact to the general public. As a result, if the investor announces nothing (i.e., $m_j = \varnothing$), the public cannot directly discern whether the investor did not conduct a visit or she did conduct a visit but chose not to release it. This treatment is similar to what is observed in standard discretionary disclosure models (e.g., Dye, 1985); although, in the Dye model, disclosure is about information per se while, in our setting, disclosure is about information acquisition.

Under mandatory disclosure, whether the investor has conducted a site visit must be disclosed to the general public. If investor j conducts a site visit, her message is $m_j = SV$; again, observing $m_j = SV$, other agents understand that $h_j \geq \bar{h}$. If investor j does not conduct any visits, then this fact is mandatorily disclosed to the public, and the investor's message is denoted by $m_j = NV$. Observing $m_j = NV$, other agents understand that $h_j \leq \bar{h}$ and form beliefs accordingly.

In our site-visit context, the threshold $\bar{h}$ is an exogenous parameter that captures the basic information advantage that an investor can achieve by conducting a site visit. This feature is notably different from classic discretionary disclosure models (e.g., Dye, 1985), in which the disclosure threshold is endogenous and a key object of interest. The value of $\bar{h}$ can be determined by firm characteristics. For example, site visits to firms with more tangible assets (e.g., manufacturing companies) may be associated with a higher $\bar{h}$ because the first-hand information gained is more valuable. In addition to the cross-sectional interpretation, we can also associate $\bar{h}$ with

[12] To illustrate our theory most transparently, we assume that site visits are more advanced than all other methods of information acquisition, which is a reasonable approximation of reality. We can also accommodate the possibility that some methods are more advanced than site visits. In such an extension, disclosing site visits would reveal that information precision belongs to a union of several intervals, which significantly complicates our analysis. Although our analysis does not consider this possibility, we believe that the key economic forces, such as the pricing and competition effects, continue to play an important role in this extended setting.

different time periods; for instance, in periods surrounding financial disclosures, site visits can be more valuable since they may offer investors opportunities to meet with firm management; thus, visits conducted during those periods can also feature a higher $\bar{h}$.

In Section S2 of the online appendix, we consider another way to model the site-visits context. In this alternative setup, all information acquisition is viewed as site visits, and an investor can decide which site visits to disclose under voluntary disclosure. Upon observing the disclosure, the general public knows that the investor has visited the firm at least these times, thereby forming the lower bound of the investor's information precision. That is, the disclosure threshold is pinned down endogenously. We show that the pricing and competition effects remain robustly the primary forces in determining investors' disclosure and information-acquisition behavior, and all the results in the main text continue to hold qualitatively.

In this site-visit context, we must also explicitly specify off-equilibrium beliefs. We propose a specification in the sense of subgame perfection and provide the details in Appendix B. In this appendix, we also provide the formal equilibrium definitions for the voluntary and mandatory disclosures of site visits. Although the complete equilibrium definitions include information precision levels, message functions, belief functions, trading strategies, and the pricing rule, what really matters for allocations is information precision as well as the resulting trading and pricing rules. Thus, when two equilibria entertain the same information-acquisition outcome and only differ in message or belief functions, we consider these two equilibria effectively the same. Also, when an investor is indifferent between disclosure and non-disclosure, we assume that she chooses non-disclosure, which would be strictly optimal if disclosure involves even an arbitrarily small cost.

3.3 Disclosure of Site Visits: Monopolistic Investor

3.3.1 Voluntary Disclosure by Monopolistic Investor

Similar to Section 2.1, only the pricing effect prevails in the monopolistic setting. Specifically, if the monopolistic investor could use disclosure to influence the market maker's belief $\hat{h}$, she would always want the market maker to hold the lowest $\hat{h}$. Taking the partial derivative of the investor's expected profit $\pi(h, \hat{h})$ in equation (8) with respect to $\hat{h}$, we have[13]

$$\frac{\partial \pi(h, \hat{h})}{\partial \hat{h}} = -\frac{h\sigma_u}{4(1+h)\hat{h}^2}\sqrt{\frac{\hat{h}}{1+\hat{h}}} < 0. \tag{18}$$

So, given any actual precision level h, the monopolist investor always wants $\hat{h}$ to be the lowest possible. This pricing effect is the key force driving the investor's information acquisition.

There are two cases to discuss, depending on the comparison between the precision threshold $\bar{h}$ achieved through site visits and the equilibrium precision h_s in the secret benchmark. Suppose $\bar{h} > h_s$. Without a communication technology, the investor does not conduct a site visit and achieves information precision h_s, and the market maker's belief $\hat{h}$ is also h_s. If voluntary disclosure is allowed, would the investor deviate by conducting a site visit and disclosing? The answer is negative due to two forces.[14] First, holding constant the market maker's belief, deviating from h_s is costly since h_s is the investor's optimal choice given the market maker's belief h_s. Second, disclosure raises the market maker's belief by revealing that the investor's information precision is at least $\bar{h}$, which is higher than h_s. This belief adjustment further harms the investor via the pricing effect. Taken together, the investor still chooses precision h_s under the voluntary disclosure setting when $\bar{h} > h_s$.

Now suppose $\bar{h} < h_s$. In this case, voluntary disclosure is ineffective in influencing the market maker's belief. Specifically, in the absence of a communication technology, the investor optimally conducts a site visit to achieve precision h_s. The market maker correctly infers that the investor's precision is h_s, which exceeds the threshold $\bar{h}$. Voluntary disclosure would reveal to the market

[13]Note that the partial derivative $\frac{\partial \pi(h,\hat{h})}{\partial \hat{h}}$ in equation (18) is essentially the pricing effect $\frac{\partial \pi}{\partial \lambda}\frac{\partial \lambda}{\partial h}$ in equation (11), where $\hat{h} = h$ in the overt benchmark economy. Specifically, by equations (6)–(8), in the monopolistic setting, $\hat{h}$ affects profit π only through λ (i.e., $\pi(h, \hat{h}) = \pi(h, \lambda(\hat{h}))$) and thus, $\frac{\partial \pi(h,\hat{h})}{\partial \hat{h}} = \frac{\partial \pi}{\partial \lambda}\frac{\partial \lambda}{\partial \hat{h}}$.

[14]We can show that, if the investor deviates to conduct a visit and disclose, she will acquire precision $\bar{h}$. We can further show that, $\pi(\bar{h}, \bar{h}) < \pi(h_s, h_s)$ when $\bar{h} > h_s$. In addition, we can break down $\pi(\bar{h}, \bar{h}) - \pi(h_s, h_s)$ as follows:

$$\pi(\bar{h}, \bar{h}) - \pi(h_s, h_s) = \underbrace{\pi(\bar{h}, h_s) - \pi(h_s, h_s)}_{<0} + \underbrace{\pi(\bar{h}, \bar{h}) - \pi(\bar{h}, h_s)}_{<0 \text{ iff } \bar{h} > h_s}. \tag{$*$}$$

The first and second terms of equation ($*$) correspond respectively to the two forces mentioned in the text. If the investor deviates to conduct a visit but does not disclose it, then only the first force prevails since the market maker's belief remains unchanged; obviously, this kind of deviation is also suboptimal.

maker the correctly inferred fact that the investor's precision is higher than $\bar{h}$ and, thus, the market maker's belief remains unchanged upon disclosure. Since the market maker's belief stays fixed at h_s and independent of the investor's messages, the investor effectively becomes secret in her information acquisition. In consequence, the equilibrium information precision under voluntary disclosure is again h_s in the case of $\bar{h} < h_s$,

Lemma 4 summarizes the equilibrium when a monopolistic investor has the option to voluntarily disclose her site visit.

Lemma 4 (Voluntary disclosure – Monopolistic investor). *In the monopolistic investor setting with the voluntary disclosure of site visits, there exists a unique equilibrium in which the following holds:*

(1) *The investor acquires information of precision h_s, where h_s is the unique solution to equation (9).*

(2) *The investor conceals her information precision and effectively engages in secret information acquisition; that is, the message function is $m(h) = \varnothing$.*

(3) *The market maker's belief function is $\hat{h}(\varnothing) = h_s$ (on equilibrium); $\hat{h}(SV) = h_s$ (off equilibrium) when $h_s > \bar{h}$, and $\hat{h}(SV) = \bar{h}$ (off equilibrium) when $h_s < \bar{h}$.*

(4) *The market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with $\lambda = \frac{\sqrt{\hat{h}}}{2\sigma_u\sqrt{1+\hat{h}}}$, and the investor's trading rule is $\tilde{x} = \alpha\tilde{y}$ with $\alpha = \frac{h}{2(1+h)\lambda}$.*

3.3.2 Mandatory Disclosure by Monopolistic Investor

Mandatory disclosure explicitly reveals whether or not an investor has conducted a site visit. Thus, when the investor switches from conducting no visits to conducting a visit, or vice versa, the market maker's beliefs about the investor's precision always change accordingly. This feature of beliefs under mandatory disclosure can differ from that under voluntary disclosure, and when this difference in belief features across the two disclosure settings indeed arises, the investor's information-acquisition incentives are altered.

Again, consider two cases. When $\bar{h} > h_s$, the market maker's beliefs change with the investor's information choice in the same way under both disclosure settings. Specifically, as discussed in Section 3.3.1, under voluntary disclosure, when the investor conducts no visits, the market maker holds belief h_s; when the investor deviates to acquire information via a visit and discloses, the market maker raises her belief above h_s. Under mandatory disclosure, the market maker's beliefs exhibit the same pattern. As a result, the investor faces the same trade-off in acquiring information across the two disclosure settings, which in turn implies that the equilibrium information precision is the same across the two settings.

Conversely, when $\bar{h} < h_s$, under voluntary disclosure, regardless of whether or not the investor conducts a visit, the market maker thinks that $\bar{h}$ is so low that the investor must've conducted a visit to achieve precision h_s. By contrast, under mandatory disclosure, the investor's decision regarding site visits always affects the market maker's beliefs. That is, if the investor conducts a visit, the market maker knows it through mandatory disclosure and forms belief h_s, which is higher than $\bar{h}$. If the investor does not conduct a visit, this fact will also be revealed to the market maker, who then observes that the investor's precision cannot exceed $\bar{h}$, thereby lowering the market maker's belief. So, deviating from conducting a visit to not conducting a visit brings two opposing effects on the investor's payoff: On one hand, holding constant the market maker's belief, the deviation is suboptimal. On the other, the deviation lowers the market maker's belief, which can benefit the investor through the pricing effect (see equation (18)).[15] When this second effect dominates, the investor will deviate and avoid conducting site visits that she would've done in the absence of mandatory disclosure.

We summarize the above results in the following lemma:

Lemma 5 (Mandatory disclosure – Monopolistic investor)**.** *Let $\eta \in (0, h_s)$ denote the unique constant characterized by $\pi(\eta, \eta) = \pi(h_s, h_s)$, where $\pi(\cdot, \cdot)$ is given by equation (8). When a monopolist investor is faced with the mandatory disclosure of site visits, there exists a unique equilibrium in*

[15] The two effects parallel the two forces described in Footnote 14. There, the two terms in the decomposition equation ($*$) are both negative while, here, they have opposite signs (i.e., the first term is still negative, while the second term becomes positive).

which the following holds:

(1) *When $\bar{h} \leq \eta$, the investor conducts a site visit and acquires information of precision h_s. When $\eta < \bar{h} \leq h_s$, the investor does not conduct a site visit and acquires information of precision $\bar{h}$. When $h_s < \bar{h}$, the investor does not conduct a site visit and acquires information precision h_s.*

(2) *When $h_s > \bar{h}$, the market maker's belief function is $\hat{h}(SV) = h_s$ and $\hat{h}(NV) = \bar{h}$. When $h_s < \bar{h}$, the market maker's belief function is $\hat{h}(SV) = \bar{h}$ and $\hat{h}(NV) = h_s$.*

(3) *The market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with $\lambda = \frac{\sqrt{\hat{h}}}{2\sigma_u\sqrt{1+\hat{h}}}$, and the investor's trading rule is $\tilde{x} = \alpha\tilde{y}$ with $\alpha = \frac{h}{2(1+h)\lambda}$.*

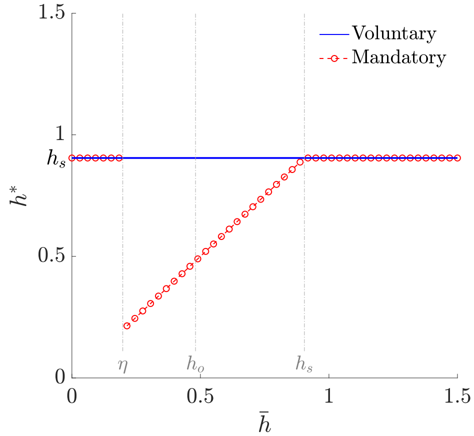
In Panel (a) of Figure 3, we plot the investor's equilibrium information precision against $\bar{h}$ under voluntary disclosure (solid line) and mandatory disclosure (dashed circled line). We find that mandatory disclosure changes the investor's information-acquisition behavior when $\eta < \bar{h} < h_s$. In this parameter range, equilibrium under voluntary disclosure dictates that the investor secretly conducts a site visit to acquire h_s. However, under mandatory disclosure, the investor decides not to conduct a site visit and, instead, exhausts the limits of other information-acquisition methods to enjoy the benefit of improved market liquidity caused by the pricing effect. We revisit this point in greater detail in Section 4.1 during discussions of policy implications.

3.4 Disclosure of Site Visits: Oligopolistic Investors

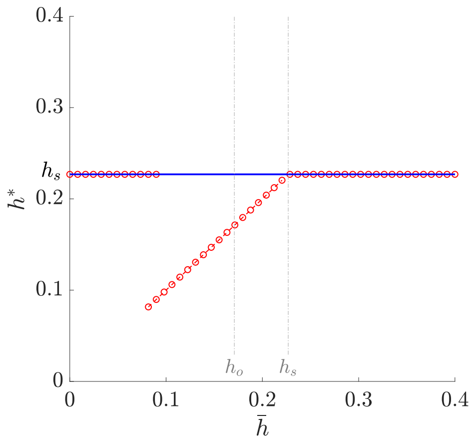
In this setting with oligopolistic investors, we focus on symmetric equilibrium in which $h_j^* = h^*$ $\forall j$ and present our results mainly in terms of h^*. Characterizing the equilibrium for this setting is quite complex; so, we delegate the formal characterization to Lemmas B1 and B2 in Appendix B and only summarize the main points in the following subsections.

3.4.1 Voluntary Disclosure by Oligopolistic Investors

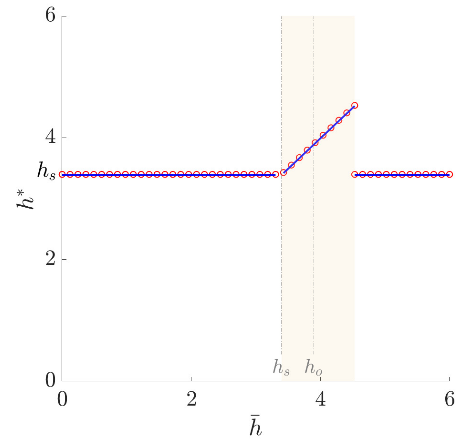
The behavior of equilibrium information precision h^* in the oligopolistic setting depends on the value of c/σ_u. Panels (b1) and (b2) of Figure 3 graphically plot the equilibrium information pre-



(a) Monopoly



(b1) Duopoly with high c/σ_u



(b2) Duopoly with low c/σ_u

This figure plots investors' equilibrium information precision h^* against $\bar{h}$ under a monopoly (Panel (a)) and duopoly (Panels (b1) and (b2)). In each panel, the solid and dashed circled lines depict the economies of voluntary and mandatory disclosure, respectively. The shaded area in Panel (b2) indicates the region in which disclosure is the dominant strategy for duopolistic investors in the voluntary disclosure economy. The constants h_s and h_o indicate the equilibrium information precision in the secret and overt benchmark economies, given by (12) and (13), respectively. The constant η is characterized by $\pi(\eta, \eta) = \pi(h_s, h_s)$. The parameter values are $\sigma_u = 1$ in all panels; $c = 0.2$ in Panel (a), $c = 0.5$ in Panel (b1), and $c = 0.015$ in Panel (b2).

Figure 3: Voluntary versus mandatory disclosure of site visits

cision h^* for high and low values of c/σ_u, respectively.

As explained in Section 2.2, when c/σ_u is relatively high, investors end up with coarse information and behave like local monopolists of their own information, rendering the oligopolistic setting like the monopolistic setting. Formally, using equations (14) through (16) in Section 2.2, we can express investor j' expected net trading profits π_j as functions of $(h_j, \hat{h}_j; h^*)$, where h_j is investor j's information precision, $\hat{h}_j$ is other agents' belief about h_j, and h^* is the equilibrium information precision chosen by other $J-1$ investors. We can show that[16]

$$\lim_{c/\sigma_u \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} < 0. \tag{19}$$

That is, when c/σ_u is high, raising other agents' beliefs $\hat{h}_j$ surrounding investor j's information precision harms investor j. This condition is similar to condition (18) in the monopolistic setting and, hence, the investors' information-acquisition incentives are similar to those in the monopolistic setting. As a result, like Lemma 4, oligopolistic investors' information acquisition is effectively secretive when c/σ_u is high.

By contrast, when c/σ_u is low, investors end up with highly accurate information and compete fiercely with one another. This competition effect implies that raising other agents' beliefs $\hat{h}_j$ about investor j's information precision benefits investor j. Formally, the derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ with respect to $\hat{h}_j$ becomes positive:

$$\lim_{c/\sigma_u \to 0} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} > 0. \tag{20}$$

The competition effect can encourage investors to conduct site visits and disclose these events to make their rivals trade less aggressively.

To see this point, consider the case of $\bar{h} > h_s$. Without a communication technology, investors do not conduct site visits and acquire precision h_s. Deviating and conducting a visit makes investor j face the following trade-off: On one hand, deviating to a suboptimal precision level harms her, with other things being equal (i.e., λ and α_{-j} are fixed); on the other hand, conducting and disclosing a site visit can inform her rivals of her being very informed so they will trade less aggressively (i.e., α_{-j} decreases), benefiting investor j.[17] When the second effect dominates, the investor deviates and conducts a site visit, as captured by the middle region of Panel

[16]Similar to (18), the partial derivative $\frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j}$ is essentially a combination of the pricing effect and the competition effect in equation (17) (where $\hat{h}_j = h_j$ in the overt benchmark economy), since by equations (14)–(16), belief $\hat{h}_j$ affects profit π_j through λ (pricing effect) and α_{-j} (competition effect).

[17]These two forces resemble those discussed in Footnote 14; see the decomposition equation ($*$). There, both forces are negative; here, the first term is still negative, while the second term becomes positive, reflecting the competition effect.

(b2) of Figure 3.

3.4.2 Mandatory Disclosure by Oligopolistic Investors

Again, as explained in Section 2.2, when c/σ_u is high, oligopolistic investors behave like monopolists, which explains why Panels (a) and (b1) of Figure 3 look similar. One additional result shown in Panel (b1) is that, due to investor coordination, multiple equilibria can exist under the oligopolistic setting. For instance, in Panel (b1) of Figure 3, when $0.082 < \bar{h} < 0.09$, both $h^* = h_s$ (i.e., site visits) and $h^* = \bar{h}$ (i.e., no visits) can be sustained in equilibrium.

When c/σ_u is low, the competition effect makes information acquisition under mandatory disclosure coincide with that under voluntary disclosure (i.e., the dashed circled line overlaps with the solid line in Panel (b2) of Figure 3). Intuitively, then, an investor seeks to reveal to other agents that she has conducted a visit, a benefit that can be equally achieved by either voluntary or mandatory disclosure.

4 Regulatory and Empirical Implications

In this section, we first discuss the implications of corporate site visits, which follow directly from our analysis in Section 3. We then extend our discussions broadly to other contexts in which information-acquisition observability is relevant and explore how the results we delivered in Sections 2 and 3 can be used to understand these extended situations.

4.1 Implications for Site Visits

Incidence of Voluntary Disclosure of Site Visits. As mentioned in Section 3.1, many financial institutions voluntarily disclose their site visits or private meetings with firms in real-life practice. Our theory predicts instances of such disclosure. Taking Lemmas 4 and B1 (or the solid lines in Figure 3), we have the following proposition:

Proposition 3 (Incidence of voluntary disclosure of site visits)**.** *(1) In the monopolistic or oligopolistic-investor setting with sufficiently high c/σ_u, investors never voluntarily disclose their site visits in the presence of arbitrarily small disclosure costs. (2) In the oligopolistic-investor setting with sufficiently low c/σ_u, investors voluntarily disclose their site visits if $h_s < \bar{h} \leq h_o$.*

In Figure 3, we use the shaded area to highlight the region in which voluntary disclosure is the dominant strategy in equilibrium. Consistent with Proposition 3, voluntary disclosure arises only in Panel (b2) of Figure 3 (i.e., duopoly with low c/σ_u). As explained in Section 3.4.1, investors are driven to voluntarily disclose their site visits by the competition effect, which is strong when J is large and c/σ_u is low. This result is testable since the empirical proxies for J and c are readily available. For instance, we can directly use the number of funds that are trading a company to measure J and then, follow the literature to infer information-acquisition cost c based on the homogeneity and accuracy of analysts' earnings forecasts.[18] Proposition 3 also predicts that voluntary disclosure is more likely when $\bar{h}$ takes intermediate values. As discussed in Section 3.2, we can ascertain the value of $\bar{h}$ by linking it to firms with different levels of tangible assets or one firm in different periods. Equipped with these proxies for $\bar{h}$, we can, for example, include a quadratic term, to test the non-monotonic prediction on $\bar{h}$.

Effectiveness and Consequence of Mandatory Disclosure of Site Visits. By comparing equilibria under voluntary and mandatory disclosure, we can gauge the effectiveness and consequences of mandating disclosure of site visits. If investors are willing to disclose site visits on a voluntary basis, mandatory disclosure is purely redundant. By contrast, if investors conceal their site visits, then mandatory disclosure can significantly alter equilibrium outcomes. Using Lemmas 4, 5, B1, and B2 (or by comparing the solid lines with the dashed, circled lines in Figure 3), we obtain the following proposition:

[18] Duchin, Matsusaka, and Ozbas (2010) use analyst-forecast dispersion and error to measure the information-acquisition cost. Intuitively, "(a) lack of consensus among analysts (high standard deviation) suggests it is difficult for outsiders to become informed about the firm...Large forecast errors indicate a greater difficulty of becoming informed" (Duchin et al., 2010, p.202). Thus, both high analyst-forecast dispersion and error imply a high information-acquisition cost.

Proposition 4 (Effectiveness and consequence of mandatory disclosure of site visits)**.** *(1) In the monopolistic investor setting, mandatory disclosure of site visits is effective if and only if* $\eta < \bar{h} \leq h_s$*. (2) In the oligopolistic investor setting, mandatory disclosure of site visits is effective when* c/σ_u *is sufficiently high and* $h_o < \bar{h} \leq h_s$*. (3) When effective, mandatory disclosure causes investors to acquire less precise information and improves their payoff and market liquidity but worsens market efficiency.*

In Figure 3, we observe that, in terms of information acquisition, mandatory disclosure differs from voluntary disclosure in Panels (a) and (b1), consistent with Parts (1) and (2) of Proposition 4. As discussed in Section 3, the difference is driven by the pricing effect: Mandatory disclosure allows investors to inform the market maker that they have not conducted site visits (which they would've conducted in the absence of mandatory disclosure), which allays the market maker's adverse-selection concern. As a result, the equilibrium features less private information and higher market liquidity, confirming Part (3) of Proposition 4.

Part (3) also suggests that mandatory disclosure can weakly Pareto-improve the well-being of all market participants. Specifically, both rational investors and noise traders are better off while the market maker always makes zero profits. Here, we follow the microstructure literature (e.g., Easley, O'Hara, and Yang, 2016) and use the expected loss $\lambda\sigma_u^2$ to negatively measure noise traders' welfare to capture the idea that they are better off if they can realize their liquidity needs at a lower expected opportunity cost. This Pareto-improvement may appear surprising since one would expect the financial market to be a zero-sum game: It *is*, but only *after* information acquisition (i.e., taking the information-acquisition cost as sunk). In our setting, investors are better off precisely because they save the information-acquisition cost under mandated disclosure.

Finally, we warn against the dark side of regulated mandatory disclosure, which is its ability to lower price efficiency. This result is consistent with recent empirical evidence provided by Bowen et al. (2020), who find that the introduction of SZSE's mandatory, near-real-time disclosure of site visits has reduced the information efficiency of SZSE-listed firms. This consequence may be undesirable if price efficiency has real effects.[19]

4.2 Regulation Fair Disclosure

In this subsection, we discuss how our analyses in Sections 2 and 3 can shed new light on Reg FD. This regulation governs all investors' information-acquisition activities so long as the information comes from direct interactions with the firms; site visits are certainly one example of activities regulated by Reg FD. In the U.S., Reg FD was passed by the Securities and Exchange Commission (SEC) in October 2000 and designed to reduce selective disclosure of material information. Specifically, Reg FD requires U.S. public companies to make the same information available across all investors rather than to disclose selectively to a privileged few, such as securities analysts and institutional investors close to the company.

Reg FD effectively improves the observability of information-acquisition activities by investors in that while it does not prohibit private meetings, it imposes disclosure requirements on this communication. Specifically, Reg FD requires that whenever a firm has private meetings with select investors, it must disclose the occurrence of this event to the public (SEC, 2000).[20] To the extent that firms care about their large investors and often engage these investors in private meetings, the market can grasp a good idea of who participates in these forthcoming meetings from the firm's disclosure.

We can broadly connect Reg FD with our analysis as follows. Prior to Reg FD, meetings between a public firm and a select few investors were private and their disclosure was voluntary. This scenario can be approximated by the secret benchmark in Section 2 or the voluntary disclosure setting in Section 3. As we argue, Reg FD improves information-acquisition observability and, so, its implementation is akin to a switch from the secret to overt benchmark in Section 2, or a switch from voluntary to mandatory disclosure in Section 3. Based on Figures 2 and 3, our

[19]For instance, company managers may learn information from asset prices to guide real decisions; see the review article by Bond, Edmans, and Goldstein (2012).

[20]For instance, in its 8-K report filed on January 8, 2019, Simpson Manufacturing Company announced that its representatives would give a presentation to certain investors (See https://www.simpsonmfg.com/docs/Form-8-K-Item-7.01-FD-Disclosure-1.08.2019.pdf).

analysis suggests that the implementation of Reg FD will decrease information production and market efficiency but increase market liquidity for firms with a small number J of investors or a high ratio c/σ_u of the information-acquisition cost to noise trading.

4.3 Other Implications

The observability of information acquisition is also relevant in the contexts of social networks and the geography of investors. For instance, Cohen, Frazzini, and Malloy (2008) and Chen et al. (2020) find that institutional managers gain informational advantages through educational commonalities with senior corporate executives. Investment companies are required to disclose the primarily responsible portfolio management team in a relatively timely manner,[21] so the market can effectively observe the company's manager-appointment decisions and infer its potential informational advantage through such a social network.

Likewise, public firms must timely disclose the election of their directors through 8-K form in a timely manner.[22] Some directors are affiliated with institutional investors. Hence, to the extent that such a board-investment-company network is valuable, the market effectively knows that the institutional investor gains private information through such a board seat.

Finally, by observing institutional investors' new location choice, the market can, again, infer their informational advantage. The literature finds that institutions know their nearby stocks well (e.g., Coval and Moskowitz, 1999, 2001; Hau, 2001; Teo, 2009). Since these institutions often list their offices on their websites or information on new locations is covered by the local press, the market's inference about investors' information-acquisition activities should be feasible.

In the above contexts, investors' information-acquisition activities become more observable through social networks and/or the geography of investors, which, again, is akin to a switch from the secret to overt benchmark in Section 2 or a voluntary to mandatory disclosure setting in Section 3. The implications are also similar; that is, information production and market effi-

[21] See https://www.sec.gov/rules/final/33-8458.htm for SEC's mandate on Disclosure Regarding Portfolio Managers of Registered Management Investment Companies.

[22] The company must disclose the election of directors (listed in Section 5 item 5.02) through 8-K, filing the form within four business days; see https://www.sec.gov/fast-answers/answersform8khtm.html.

ciency decrease but market liquidity increases for firms with a small J or a high c/σ_u. Our intention, however, is not to claim that financial institutions make appointments or location decisions primarily with strategic considerations for competition among investors or the market maker's price-setting behavior. Instead, we want to emphasize that our analysis is helpful in understanding the consequences of these decisions from the novel perspective of information-acquisition observability.

5 Conclusion

We develop a parsimonious model that allows for different degrees of observability surrounding investors' information-acquisition activities in financial markets. We find that increasing this observability gives rise to two strategic effects on information acquisition. First, the pricing effect arises from interactions between investors and the market maker, either encouraging or discouraging information acquisition depending on the number of investors and the precision level of their information. Second, the competition effect is induced by interactions amongst investors and always strengthens investors' information-acquisition incentives. The interplay of these two effects drives the implications that increasing information-acquisition observability has on equilibrium market outcomes.

We apply our theory to financial institutions' corporate site visits, an effective but often costly information-acquisition strategy. Our analysis sheds light on when investors are willing to voluntarily disclose their otherwise secretive site visits as well as on the effectiveness and consequences of mandatory disclosure. We find that when the competition effect dominates, investors are likely to voluntarily disclose their site visits. But, when the pricing effect dominates, mandatory disclosure is effective in changing equilibrium information acquisition and other market outcomes. These results are useful for policymakers when weighing the costs and benefits of mandating the disclosure of financial institutions' information-acquisition activities. Our theory can also be applied to understand the reverberating effects of Reg FD and other contexts, such as social networks and investor geography, to which information-acquisition observability is relevant.

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Appendix A: Proofs

Proof of Lemma 1 and Proposition 1

See the main text for the majority of the proof for Lemma 1. Here, we only show that there exists a unique equilibrium in both the secret and overt benchmarks. Let $g(h) = h(1+h)^3$. Since $g(0) = 0$ and $g(+\infty) = +\infty$, via intermediate value theorem, there exists a positive root of $g(h_s) = \sigma_u^2/(4c^2)$ and $g(h_o) = \sigma_u^2/(16c^2)$. Furthermore, since $g'(h) > 0$ for $h > 0$, the root of the respective equations must be unique.

For Proposition 1, since $h = h_o$ is the solution to $g(h) = \sigma_u^2/(16c^2)$, $h = h_s$ is the solution to $g(h) = \sigma_u^2/(4c^2)$, and $\sigma_u^2/(16c^2) < \sigma_u^2/(4c^2)$, we have $h_o < h_s$. Based on the discussion in the main text, $\lambda_o < \lambda_s$ and $Var(\tilde{v}|\tilde{p})_o^{-1} < Var(\tilde{v}|\tilde{p})_s^{-1}$ immediately follow.

Proof of Lemma 2

The equilibrium computation parallels Section 2, but we must take into account the interactions among investors. In a symmetric linear equilibrium, all investors choose the same precision level h^*, where $h^* = h_s$ under secret information acquisition and $h^* = h_o$ under overt information acquisition. Let other agents' beliefs about investor j's signal precision be $\hat{h}_j$.

We first consider investor k's optimal trading strategy, where $k \neq j$ and $k \in \{1, ..., J\}$. We denote investor j's trading strategy as α_j, investor k's trading strategy as α_k, all other investors' trading strategy as α_{-j}, and the market maker's pricing rule as λ. Given her information-acquisition decision $h_k = h^*$, investor k's optimal trading strategy is characterized as follows:

$$\max_{x_k} x_k \left(\frac{h^*}{1+h^*}\tilde{y}_k - \lambda \left(x_k + \alpha_j \frac{h^*}{1+h^*}\tilde{y}_k + (J-2)\alpha_{-j}\frac{h^*}{1+h^*}\tilde{y}_k \right) \right).$$

Solving it yields investor k's optimal trading strategy: $x_k = \alpha_k \tilde{y}_k$, where $\alpha_k = \frac{h^*(1-(\alpha_j+(J-2)\alpha_{-j})\lambda)}{2(1+h^*)\lambda}$.

Imposing $\alpha_k = \alpha_{-j}$ yields

$$\alpha_{-j} = \frac{h^*(1-\alpha_j\lambda)}{(2+h^*J)\lambda}. \tag{A1}$$

Similarly, investor j's trading problem is as follows:

$$\max_{x_j} x_j \left(\frac{h_j}{1+h_j}\tilde{y}_j - \lambda \left(x_j + (J-1)\alpha_{-j}\frac{h_j}{1+h_j}\tilde{y}_j \right) \right). \tag{A2}$$

Solving it yields investor j's optimal trading strategy: $x_j = \alpha_j \tilde{y}_j$, where

$$\alpha_j = \frac{h_j\left(1-(J-1)\alpha_{-j}\lambda\right)}{2(1+h_j)\lambda}. \tag{A3}$$

Combining equations (A1) and (A3) and replacing $h_j = \hat{h}_j$, we find that, based on the beliefs $\hat{h}_j$, the other investors' trading strategy is given by (16) and all other agents' conjecture about investor j's trading strategy is as follows:

$$\hat{\alpha}_j = \frac{(2+h^*)\hat{h}_j}{\left(4+2Jh^*+(4+h^*+h^*J)\hat{h}_j\right)\lambda}. \tag{A4}$$

According to (1), the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$, where

$$\lambda = \frac{(J-1)\alpha_{-j}+\hat{\alpha}_j}{\left((J-1)\alpha_{-j}+\hat{\alpha}_j\right)^2+(J-1)\frac{\alpha_{-j}^2}{h^*}+\frac{\hat{\alpha}_j^2}{\hat{h}_j}+\sigma_u^2}.$$

Replacing α_{-j} in (16) and $\hat{\alpha}_j$ in (A4) and solving for λ yields equation (15).

Next, replacing $\tilde{x}_j = \alpha_j\tilde{y}_j$ into (A2) where α_j is given by (A3), we insert (15) and (16) into (A2), take expectations, and subtract the information-acquisition cost. We, thus, obtain investor j's expected net trading profit as follows:

$$\pi_j(h_j,\hat{h}_j;h^*) = \frac{h_j(1+\hat{h}_j)^2(2+h^*)^2\sigma_u}{\left(\begin{array}{c}(1+h_j)(4+2Jh^*+\hat{h}_j(4+h^*+h^*J)) \\ \times\sqrt{\begin{array}{c}4h^*(1+h^*)(J-1)\\ +\hat{h}_j(4-3h^{*2}+4Jh^*(1+h^*))\\ +\hat{h}_j^2(4+h^*(3+J+Jh^*))\end{array}}\end{array}\right)} - c\cdot h_j. \tag{A5}$$

Under secret information acquisition, replacing $h^* = h_s$ and $\hat{h}_j = h_s$ in equation (A5) yields

$$\pi_j(h_j,h_s;h_s) = \frac{h_j(1+h_s)^2\sigma_u}{(1+h_j)(2+h_s+h_sJ)\sqrt{h_s(1+h_s)J}} - ch_j. \tag{A6}$$

Maximizing the net profit with respect to h_j and imposing $h_j = h_s$ yield the equilibrium information acquisition h_s, as characterized by equation (12). The concavity of equation (A6) rules out mixed strategies of information acquisition.

Under overt information acquisition, replacing $h^* = h_o$ and $\hat{h}_j = h_j$ in equation (A5) yields:

$$\pi_j(h_j, h_j; h_o) = \frac{h_j(1+h_j)(2+h_o)^2\sigma_u}{\left(\begin{array}{c}(4+2Jh_o+h_j(4+h_o+h_oJ)) \\ \times\sqrt{\begin{array}{c}4h_o(1+h_o)(J-1) \\ +h_j(4-3h_o^2+4Jh_o(1+h_o)) \\ +h_j^2(4+h_o(3+J+Jh_o))\end{array}}\end{array}\right)} - c \cdot h_j. \tag{A7}$$

Similarly, maximizing the net profit with respect to h_j and imposing $h_j = h_o$ yield the equilibrium information acquisition h_o, as characterized by equation (13). Again, the concavity of equation (A7) rules out mixed strategies of information acquisition.

Next, in equilibrium λ and α_{-j} are connected to h through equations (15) and (16), respectively. Setting $h_j = h_o = h$ in equations (15) and (16) yields $\lambda = \frac{\sqrt{h(1+h)J}}{\sigma_u(2+(J+1)h)}$ and $\alpha_{-j} = \sigma_u\sqrt{\frac{h}{(1+h)J}}$, respectively. With the equilibrium information precision h_s and h_o we can derive $\alpha_s, \lambda_s, \alpha_o$, and λ_o as specified in Lemma 2.

Finally, we confirm that the equilibrium information precision h_s and h_o are unique. Denote the left-hand-side (LHS) of equation (12) as $f_s(h)$. It's simple to show that $f_s(0) = 0$ and $f_s(+\infty) = +\infty$. Thus, via the intermediate value theorem, there exists a positive root of equation (12). Furthermore, since $f_s(h)' > 0$, the root of equation (12) must be unique. Similarly, denote the LHS of equation (13) as $f_o(h)$. As $f_o(0) = 0$ and $f_o(+\infty) = +\infty$, via the intermediate value theorem, there exists a positive root of equation (13). In addition, $f_o(h)$ monotonically increases in h because

$$f_o'(h) = \frac{(2+h)(2+h+hJ)^3 \cdot \Omega(h)}{\left((6J^2-J-3)h^2+(4J^2+10J-8)h+8J-4\right)^3} > 0,$$

where $\Omega(h) = 4h^5(6J^3+5J^2-4J-3) + h^4(66J^3+119J^2-16J-69) + 4h^3(16J^3+53J^2+25J-36) + 4h^2(6J^3+43J^2+46J-34) + 16h(4J^2+7J-4) + 16(2J-1)$. The inequality follows because we can prove that the coefficients of all polynomial terms regarding h in $\Omega(h)$ are positive for $J \geq 2$. For instance, we denote term h^5's coefficient as $\Omega_{o5}(J)$, that is, $\Omega_{o5}(J) \equiv 4(6J^3+5J^2-4J-3)$. Because $\Omega_{o5}'(J) = 8(9J^2+5J-2) > 0$ for all $J \geq 2$, we must have $\Omega_{o5}(J) \geq \Omega_{o5}(2) = 44 > 0$ for all $J \geq 2$. Similarly, we can check all coefficients, and $f_o'(h) > 0$

immediately follows. Therefore, the real root of equation (13) is unique.

Proof of Lemma 3

Investor j's expected profit is given by equation (14), where λ and α are given by equations (15) and (16), respectively. Taking the derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ with respect to h_j yields (17). Furthermore,

$$\frac{\partial \pi_j}{\partial h_j} = \frac{(1-(J-1)\alpha_{-j}\lambda)^2}{4\lambda(1+h_j)^2} - c,$$
$$\frac{\partial \pi_j}{\partial \lambda} = -\frac{\left(1-(J-1)^2\alpha_{-j}^2\lambda^2\right)h_j}{4\lambda^2(1+h_j)} < 0,$$
$$\frac{\partial \pi_j}{\partial \alpha_{-j}} = -\frac{(J-1)(1-(J-1)\alpha_{-j}\lambda)h_j}{2(1+h_j)} < 0.$$

Note that $\frac{\partial \pi_j}{\partial \lambda} < 0$ because equation (A3) dictates that that $1-(J-1)\alpha_{-j}\lambda > 0$.

Under secret information, $\frac{\partial \lambda}{\partial h_j} = 0$ and $\frac{\partial \alpha_{-j}}{\partial h_j} = 0$. Next, we explore overt information acquisition. Since

$$\frac{\partial \alpha_{-j}}{\partial h_j} = -\frac{\alpha_{-j}^3(2+h^*)^2(2+3h_j)}{2h^{*2}\sigma_u^2(2+h_j)^3} < 0,$$

the competition effect is always positive, i.e., $\frac{\partial \pi_j}{\partial \alpha_{-j}}\frac{\partial \alpha_{-j}}{\partial h_j} > 0$.

We next examine the pricing effect when information precision is the same across investors. Given the other investors' information precision h^*, we find that

$$\frac{\partial \lambda}{\partial h_j} = -\frac{(2+h^*)\Big(\big((J-3)h^{*2}-2(J+3)h^*-8\big)h_j + \big(2(3J-4)h^{*2}+4h^*(J-3)-8\big)\Big)}{2\lambda\sigma_u^2\left(h_j(h^*J+h^*+4)+2h^*J+4\right)^3}. \tag{A8}$$

At $h_j = h^*$, equation (A8) can be simplified as the following:

$$\frac{\partial \lambda}{\partial h_j}|_{h_j=h^*} = \frac{2-(J-3)h^*}{2\sqrt{h^*(h^*+1)J}(h^*J+h^*+2)^2\sigma_u}.$$

When $J \leq 3$, $\frac{\partial \lambda}{\partial h_j}|_{h_j=h^*} > 0$, so that the pricing effect is negative, i.e., $\frac{\partial \pi_j}{\partial \lambda}\frac{\partial \lambda}{\partial h_j}|_{h_j=h^*} < 0$. When $J > 3$, $\frac{\partial \lambda}{\partial h_j}|_{h_j=h^*} > 0$ if and only if $h^* < \frac{2}{J-3}$; thus, the pricing effect is negative if and only if $h^* < \frac{2}{J-3}$.

Proof of Proposition 2

We now compare the equilibrium information acquisition across overt and secret information acquisition. First, we show that, when $c \to 0$, $h_o > h_s$. Note that, when $c \to 0$, $h_o \to \infty$ and $h_s \to \infty$. Equations (12) and (13) can be simplified as follows: $\frac{c}{\sigma_u} = \frac{1}{\sqrt{J}(J+1)h_s^2 + o(h_s^2)}$ and $\frac{c}{\sigma_u} = \frac{(6J^2-J-3)h_o^2+o(h_o^2)}{2J\sqrt{J}(J+1)^2h_o^4+o(h_o^4)}$, respectively. It follows that

$$h_o = \frac{\sigma_u}{c} \cdot \frac{6J^2-J-3}{2J\sqrt{J}(J+1)^2} + o\left(\frac{1}{c}\right),$$
$$h_s = \frac{\sigma_u}{c} \cdot \frac{1}{\sqrt{J}(J+1)} + o\left(\frac{1}{c}\right),$$

where $\frac{6J^2-J-3}{2J\sqrt{J}(J+1)^2} > \frac{1}{\sqrt{J}(J+1)}$ for any $J \geq 2$. Therefore, when $c \to 0$, $h_o > h_s$.

Second, consider the case where $c \to \infty$, which leads to $h_o \to 0$ and $h_s \to 0$. Again, equations (12) and (13) can be simplified as follows: $\frac{c}{\sigma_u} = \frac{1}{2\sqrt{h_o J}+o(\sqrt{h_s})}$ and $\frac{c}{\sigma_u} = \frac{8J-4+o(1)}{16J\sqrt{h_o J}+o(\sqrt{h_o})}$ respectively. It follows that

$$\sqrt{h_o J} = \frac{2J-1}{4J} \cdot \frac{\sigma_u}{c} + o\left(\frac{1}{c}\right), \tag{A9}$$
$$\sqrt{h_s J} = \frac{\sigma_u}{c} + o\left(\frac{1}{c}\right). \tag{A10}$$

Using the fact that $\frac{\sigma_u}{c} + o\left(\frac{1}{c}\right) > \frac{2J-1}{4J} \cdot \frac{\sigma_u}{c} + o\left(\frac{1}{c}\right)$, we can show that $h_s > h_o$. Therefore, we prove that when $c \to \infty$, $h_s > h_o$.

Next, consider the function $\Delta(c) = h_o - h_s$. We wish to find out the roots of $\Delta(c) = 0$. Straightforward calculations suggest that $\Delta(c) = 0$ holds if and only if $h_o = h_s = \zeta_0$, where ζ_0 is given in Proposition 2. And this corresponds to a single solution $c = \zeta\sigma_u$, where ζ is given in Proposition 2. Using the intermediate value theorem, we prove that when $c/\sigma_u > \zeta (c/\sigma_u < \zeta)$, $h_s > h_o (h_s < h_o)$. The remainder of the proof regarding the comparison of market efficiency and market liquidity follows immediately from discussions in the main text.

Proof of Lemmas 4 and 5

As Lemmas 4 and 5 are closely related, we prove them together. Given the investor's information precision h and the market maker's belief $\hat{h}$, we can still show that the investor's date-0 expected net profit function $\pi(h, \hat{h})$ is given by (8). We first present the following auxiliary lemma that we need for the subsequent proof.

Lemma A1. *The following facts hold under monopoly:*

(1) $\frac{\partial \pi(h,\hat{h})}{\partial \hat{h}} < 0$*, i.e., given any* h*, the investor wants the market maker's belief the lowest possible.*

(2) $\frac{\partial \pi(h,h)}{\partial h}|_{h=h_o} = 0$ *and* $\frac{\partial^2 \pi(h,h)}{\partial h^2} < 0$*. That is,* $\pi(h,h)$ *is single peaked at* $h = h_o$*.*

(3) $\frac{\partial \pi(h,h_s)}{\partial h_s}|_{h=h_s} = 0$ *and* $\frac{\partial^2 \pi(h,h_s)}{\partial h_s^2} < 0$*. That is,* $\pi(h,h_s)$ *is single peaked at* $h = h_s$*.*

(4) *Given that* $\hat{h} \leq h_s$*,* $\pi(h, \hat{h})$ *increases in* h *when* $h \leq h_s$*.*

Proof. Parts (1)–(3) of Lemma A1 are straightforward so we skip their proofs. Part (4) holds because $\frac{\partial \pi(h,\hat{h})}{\partial h} = -c + \frac{\sigma_u}{2(1+h)^2}\sqrt{\frac{1+\hat{h}}{\hat{h}}} \geq -c + \frac{\sigma_u}{2(1+h_s)^{3/2}\sqrt{h_s}} = 0$, where the inequality follows because $\frac{\partial \pi(h,\hat{h})}{\partial h}$ decreases in both h and $\hat{h}$, and $h, \hat{h} \leq h_s$, and the last equality follows the definition of h_s. □

For the rest of the proof, we focus on the mandatory disclosure setting first (i.e., Lemma 5) and then the voluntary disclosure setting (i.e., Lemma 4). For each setting, we follow these steps to characterize the equilibrium: First, we categorize discussions into four cases based on (i) whether $h^* \leq \bar{h}$ or $h^* \geq \bar{h}$ and (ii) whether $\bar{h} \leq h_s$ or $\bar{h} > h_s$. The comparison of h^* and $\bar{h}$ matters because it determines whether the investor conducts a site visit in equilibrium, while the comparison of $\bar{h}$ and h_s matters to the market maker's belief. Second, in each case, we characterize the candidate equilibria and then examine the conditions under which such candidate equilibria can, indeed, be sustained in equilibrium. By discussing all four cases, we can characterize all possible equilibria.

Mandatory Disclosure (Lemma 5)

Case 1. $\bar{h} \leq h_s$ and $h^* \leq \bar{h}$. The condition of $h^* \leq \bar{h}$ implies that the candidate equilibrium features no site visits. Since the equilibrium precision h^* maximizes the investor's profit across all precision levels, it must also maximize the investor's profit subject to $h \leq \bar{h}$. That is, $h^* = \arg\max_{h \leq \bar{h}} \pi(h, \hat{h})$, where $\pi(h, \hat{h}) = \frac{h\sigma_u}{2(1+h)}\sqrt{\frac{1+\hat{h}}{\hat{h}}} - c \cdot h$. The Lagrangian function is

$$\mathcal{L}_1 = \frac{h\sigma_u}{2(1+h)}\sqrt{\frac{1+\hat{h}}{\hat{h}}} - c \cdot h + \mu(\bar{h} - h), \tag{A11}$$

where $\mu \geq 0$ is the Lagrangian multiplier. Taking the derivative with respect to h and then imposing consistent beliefs on the equilibrium path $\hat{h} = h^*$ yields

$$\frac{\sigma_u}{2(1+h^*)^{3/2}\sqrt{h^*}} - (c + \mu) = 0. \tag{A12}$$

The complementary slackness conditions are $\mu \geq 0$, $h^* \leq \bar{h}$, and $\mu(\bar{h} - h^*) = 0$.

If $\mu = 0$, then equation (A12) yields the unique solution $h^* = h_s$, where h_s is given by (9). Based on the constraint $h^* \leq \bar{h} \leq h_s$, this equilibrium can be sustained only when $\bar{h} = h_s$, which can be combined in the following case in which $\mu > 0$.

If $\mu > 0$, the constraint must be binding, so the candidate equilibrium is $h^* = \bar{h}$. Given that $\frac{\sigma_u}{2(1+h_s)^{3/2}\sqrt{h_s}} = c$, $\bar{h} \leq h_s$, and $\frac{\sigma_u}{2(1+h)^{3/2}\sqrt{h}}$ decreases in h, we can find a $\mu \geq 0$ so that $\bar{h}$ solves equation (A12). That is, precision $\bar{h}$, indeed, solves $\max_{h \leq \bar{h}} \pi(h, \bar{h})$, and the investor's payoff is $\pi(\bar{h}, \bar{h})$. To support $\bar{h}$ as an equilibrium, we must ensure that the investor has no incentives to deviate to $h \geq \bar{h}$ (i.e., site visit). If the investor deviates, then the message function reveals to the market maker that the investor has conducted a site visit; so, we can use equation (B3) to compute the market maker's consistent off-equilibrium belief as $\hat{h}(SV) = h_s$. Thus, after deviating, the investor makes profit $\pi(h_s, h_s)$, rendering $\bar{h}$ an equilibrium if and only if $\pi(\bar{h}, \bar{h}) \geq \pi(h_s, h_s)$. We can show that this holds true if and only if $\eta < \bar{h} \leq h_s$, where $\eta < h_o$ and $\pi(\eta, \eta) = \pi(h_s, h_s)$, by using the fact $h_o < h_s$ (see Proposition 1) and Part (2) of Lemma A1. The discussion of Case 1 yields the following result:

Result A5.1. *(1) When $\eta < \bar{h} \leq h_s$, there exists an equilibrium in which the investor does not conduct a site visit and acquires information of precision $\bar{h}$ and in which the market maker's belief* *is $\hat{h}(SV) = h_s$ and $\hat{h}(NV) = \bar{h}$. (2) When $\bar{h} \leq \eta$, there does not exist an equilibrium in which the investor does not conduct a site visit.*

Case 2. $\bar{h} \leq h_s$ and $h^* \geq \bar{h}$. The condition of $h^* \geq \bar{h}$ implies that the investor conducts a site visit in this candidate equilibrium. Similar to Case 1, now h^* solves the problem $\max_{h \geq \bar{h}} \pi(h, \hat{h})$. The Lagrangian function is

$$\mathcal{L}_2 = \frac{h\sigma_u}{2(1+h)}\sqrt{\frac{1+\hat{h}}{\hat{h}}} - c \cdot h + \mu(h - \bar{h}), \tag{A13}$$

where $\mu \geq 0$ is the Lagrangian multiplier. Taking the derivative with respect to h and imposing the consistent beliefs on the equilibrium path $\hat{h} = h^*$ yields

$$\frac{\sigma_u}{2(1+h^*)^{3/2}\sqrt{h^*}} - (c - \mu) = 0. \tag{A14}$$

The complementary slackness conditions are $\mu \geq 0$, $h^* \geq \bar{h}$, and $\mu(h^* - \bar{h}) = 0$.

If $\mu > 0$, the constraint is binding and the solution is $h^* = \bar{h}$. However, based on equation (A14), $c - \mu = \frac{\sigma_u}{2(1+\bar{h})^{3/2}\sqrt{\bar{h}}} \geq \frac{\sigma_u}{2(1+h_s)^{3/2}\sqrt{h_s}} = c$, where the inequality follows $\bar{h} \leq h_s$ and the last equality follows the definition of h_s. But, when $\mu > 0$, $c - \mu < c$, which is a contradiction. In consequence, $h^* = \bar{h}$ cannot be sustained in equilibrium.

If $\mu = 0$, then equation (A14) yields the unique solution $h^* = h_s$, which is the optimal precision achieved through a site visit. The resulting profit is $\pi(h_s, h_s)$. If the investor has no incentives to deviate to no site visits (i.e., $h < \bar{h}$), h_s can be supported as an equilibrium. Upon the investor's deviation, the message NV reveals that the investor has not conducted a visit, and we can use equation (B3) to compute the market maker's belief as $\hat{h}(NV) = \bar{h}$. The resulting profit is $\pi(\bar{h}, \bar{h})$. And based on Part (2) of Lemma A1, the deviation is unprofitable (i.e., $\pi(\bar{h}, \bar{h}) \leq \pi(h_s, h_s)$) if and only if $\bar{h} \leq \eta$, so $h^* = h_s$ can be sustained in equilibrium. Summarizing Case 2, we have the following result:

Result A5.2. *(1) When $\bar{h} \leq \eta$, there exists an equilibrium in which the investor conducts a site visit and acquires information of precision h_s and in which the market maker's belief is $\hat{h}(SV) = h_s$ and $\hat{h}(NV) = \bar{h}$. (2) When $\eta < \bar{h} \leq h_s$, there does not exist an equilibrium in which the investor conducts a site visit.*

Case 3. $\bar{h} > h_s$ and $h^* \leq \bar{h}$. In this case, the candidate equilibrium features no site visits. So, h^* solves the same constrained maximization problem as in (A11). If $\mu > 0$, the constraint is binding and the solution is $h^* = \bar{h}$. However, equation (A12) shows that $c + \mu = \frac{\sigma_u}{2\sqrt{\bar{h}}(1+\bar{h})^{3/2}} < \frac{\sigma_u}{2\sqrt{h_s}(1+h_s)^{3/2}} = c$, where the inequality follows $\bar{h} > h_s$. A contradiction arises since $\mu > 0$. So, $h^* = \bar{h}$ cannot be the equilibrium.

If $\mu = 0$, the unique solution to equation (A12) is $h^* = h_s$. Will the investor deviate to $h \geq \bar{h}$ (i.e., site visits)? No. Specifically, upon the investor's deviation, the market maker updates her belief to $\hat{h}(SV) = \bar{h}$, which is computed from equation (B3), and the investor makes profit $\pi(h, \bar{h}) < \pi(h, h_s) \leq \pi(h_s, h_s)$, where the two inequalities follow Parts (1) and (3) of Lemma A1 respectively.

Result A5.3. *When $\bar{h} > h_s$, there exists an equilibrium in which the investor does not conduct a site visit and acquires information of precision h_s and in which the market maker's belief function is $\hat{h}(SV) = \bar{h}$ and $\hat{h}(NV) = h_s$.*

Case 4. $\bar{h} > h_s$ and $h^* \geq \bar{h}$. In this last case, the candidate equilibrium features site visits. The precision level h^* solves the same constrained maximization problem in (A13). If $\mu = 0$, the solution to (A14) is $h^* = h_s$. But this cannot be sustained in equilibrium since it contradicts $h \geq \bar{h} > h_s$.

If $\mu > 0$, the constraint $h \geq \bar{h}$ is binding and the solution is $h^* = \bar{h}$. However, $\bar{h}$ cannot be sustained as an equilibrium, either. Specifically, if the investor deviates to $h \leq \bar{h}$ (i.e., no site visit), the market maker holds belief $\hat{h}(NV) = h_s$ computed from equation (B3), and the investor optimally acquires information of precision h_s and makes profit $\pi(h_s, h_s) > \pi(\bar{h}, \bar{h})$, where the inequality follows $\bar{h} > h_s > h_o$ and Part (2) of Lemma A1.

Result A5.4. *When $\bar{h} > h_s$, there does not exist an equilibrium in which the investor conducts a site visit.*

In sum, Results A5.1 through A5.4 deliver Parts (1) and (2) of Lemma 5. Part (3) of Lemma 5 follows equations (5) and (6).

Voluntary Disclosure (Lemma 4)

Case 1'. $\bar{h} \leq h_s$ and $h^* \leq \bar{h}$. As discussed in Case 1, the candidate equilibrium is $h^* = \bar{h}$ and the investor is unable to disclose because she does not conduct a site visit (i.e., $h \leq \bar{h}$). Will the investor deviate to $h \geq \bar{h}$? Yes. If the investor deviates to $h \in [\bar{h}, h_s]$ and makes no disclosure, the market maker sticks to the belief $\hat{h}(\varnothing) = \bar{h}$. Then, the investor's profit becomes $\pi(h, \bar{h}) \geq \pi(\bar{h}, \bar{h})$, where the inequality follows Part (4) of Lemma A1.

Result A4.1. *When $\bar{h} \leq h_s$, there does not exist an equilibrium in which the investor does not conduct a site visit.*

Case 2'. $\bar{h} \leq h_s$ and $h^* \geq \bar{h}$. As analyzed in Case 2, the candidate equilibrium is $h^* = h_s$ and the investor is indifferent about disclosure. Given the belief $\hat{h}(SV) = \hat{h}(\varnothing) = h_s$, Part (3) of Lemma A1 dictates that the investor has no incentives to deviate to $h \leq \bar{h}$ (i.e., no site visit). Moreover, we assume that, in equilibrium, the investor does not disclose since this strategy is robust against the introduction of an arbitrarily small disclosure cost.

Result A4.2. *When $\bar{h} \leq h_s$, there exists an equilibrium in which the investor conducts a site visit and acquires information of precision h_s. The investor does not disclose: $m(h) = \varnothing$. And the market maker's belief is $\hat{h}(\varnothing) = h_s$ (on equilibrium) and $\hat{h}(SV) = h_s$ (off equilibrium).*

Case 3'. $\bar{h} > h_s$ and $h^* \leq \bar{h}$. Based on Case 3, the candidate equilibrium features no site visits and the investor acquires information of precision $h^* = h_s$ and does not disclose. Will the investor deviate to $h \geq \bar{h}$ (i.e., site visits)? No. First, by Part (1) of Lemma A1, even if the investor deviates, she has no incentive to disclose since she wants to ensure that the market maker's belief sticks to $\hat{h}(\varnothing) = h_s$. Second, given this belief and Part (3) of Lemma A1, the investor must acquire $h = h_s$.

Result A4.3. *When $\bar{h} > h_s$, there exists an equilibrium in which the investor does not conduct a site visit and acquires information of precision h_s. The investor does not disclose: $m(h) = \varnothing$. And the market maker's belief is $\hat{h}(\varnothing) = h_s$ (on equilibrium) and $\hat{h}(SV) = \bar{h}$ (off equilibrium).*

Case 4'. $\bar{h} > h_s$ and $h^* \geq \bar{h}$. Based on Case 4, the candidate equilibrium features site visits and the investor acquires information of precision $h^* = \bar{h}$ and discloses (note that, to support this candidate equilibrium, the investor must disclose). The resulting market maker's belief is $\hat{h}(SV) = \bar{h}$. However, if the investor deviates to non-disclosure, the market maker's belief becomes $\hat{h}(\varnothing) = h_s$ according to equation (B3). Since $\hat{h}(SV) > \hat{h}(\varnothing)$, by Part (1) of Lemma A1, the investor always wants to deviate to non-disclosure.

Result A4.4. *When $\bar{h} > h_s$, there does not exist an equilibrium in which the investor conducts a site visit.*

In sum, Results A4.1 through A4.4 deliver Parts (1) through (3) of Lemma 4. Part (4) of Lemma 4 follows equations (5) and (6).

Proof of Proposition 3

The proof directly follows Lemmas 4 and B1.

Proof of Proposition 4

The effectiveness of the disclosure mandate directly follows Lemmas 4 and 5 and B1 and B2, which proves Parts (1) and (2). Now, we prove Part (3). Based on Lemmas 4 and 5 and B1 and B2, conditional on the mandate being effective, investors acquire information of precision h_s under the voluntary disclosure setting whereas they acquire information of precision $\bar{h}$ under the mandatory disclosure setting. As $\bar{h} \leq h_s$, the mandate causes investors to acquire less precise information. Moreover, based on the discussion in Section 2.2, given precision level h, market efficiency is $Var(\tilde{v}|\tilde{p})^{-1} = \frac{2+h+hJ}{2+h}$, which increases in h. Thus, the implication for market efficiency immediately follows.

Regarding implications for market liquidity, based on the discussion in Section 2.2 and given precision level h, $\lambda(h) = \frac{\sqrt{h(1+h)J}}{\sigma_u(2+h(J+1))}$, which increases in h when (i) $J \leq 3$ or (ii) when $J > 3$ and $\sqrt{hJ} < \sqrt{\frac{2J}{J-3}}$. Thus, it immediately follows that the mandate improves market liquidity under the monopolistic setting (i.e., $\lambda(\bar{h}) < \lambda(h_s)$). We now examine the oligopolistic setting: When $c/\sigma_u \to \infty$, (A9) and (A10) dictates that, for a given J, $\sqrt{h_o J} \to 0$ and $\sqrt{h_s J} \to 0$, both of which lie in the region $\sqrt{hJ} < \sqrt{\frac{2J}{J-3}}$. Given that $h_o < \bar{h} \le h_s$, we have $\lambda(\bar{h}) \le \lambda(h_s)$; that is, when $c/\sigma_u \to \infty$, the mandate also improves market liquidity in the oligopolistic setting.

Finally, we examine the investors' payoff. In the monopoly setting, if $\eta < \bar{h} \le h_s$, $\pi(\bar{h}, \bar{h}) \ge \pi(h_s, h_s)$ due to Part (2) of Lemma A1. So, mandatory disclosure improves the investor's payoff. In the oligopolistic setting, when $c/\sigma_u \to \infty$ and $h_o < \bar{h} \le h_s$, investors' payoff improves upon the disclosure mandate because $\pi_j(\bar{h}, \bar{h}; \bar{h}) > \pi_j(h_s, h_s; \bar{h}) > \pi_j(h_s; h_s; h_s)$, where the first inequality follows Part (6) of Lemma B3 and the second holds since $\pi_j(h_j, h_j; h^*)$ decreases in h^*: $\frac{\partial \pi_j(h_j, h_j; h^*)}{\partial h^*} = -\frac{h_j(h_j+1)(h_j+2)(h^*+2)(J-1)\sigma_u}{2((h_j+2)h^* J + h_j(h^*+4)+4)^2} \Psi_3 < 0$, where

$$\Psi_3 = \frac{\begin{pmatrix} h_j^2 \left(h^{*2}(7J+3) + h^*(6J+26) + 24\right) \\ +h_j \left(h^{*2}(28J-6) + 8h^*(3J+5) + 40\right) + 4\left(h^{*2}(7J-4) + h^*(6J+2) + 4\right) \end{pmatrix}}{\begin{pmatrix} h_j^2 \left(h^{*2}J + h^*(J+3) + 4\right) + h_j \left(h^{*2}(4J-3) + 4h^* J + 4\right) \\ +4h^*(h^*+1)(J-1) \end{pmatrix}^{3/2}} > 0.$$

Appendix B: Additional Materials to Section 3

B1. Equilibrium Definitions in Site Visits

In our application of site visits, the messages take a binary form and we must explicitly deal with off-equilibrium beliefs. We use the monopolistic setting with mandatory disclosure to illustrate this issue and how we address it. Suppose that the equilibrium features no site visits (i.e., $h^* \le \bar{h}$). In equilibrium, the market maker observes message $m(h^*) = NV$, where $m(\cdot)$ is the message function that maps each precision level h to one message value, SV or NV (under mandatory disclosure, $m(h) = NV$ if $h \le \bar{h}$ and $m(h) = SV$ if $h \ge \bar{h}$). Since NV is on the equilibrium path, consistency implies that $\hat{h}(NV) = h^*$. Next, we must specify $\hat{h}(SV)$, which is useful in computing the investor's precision-deviation incentives on date 0. However, message SV is off-equilibrium and, so, we cannot apply Bayes rule. Therefore, we propose the following off-equilibrium belief $\hat{h}$ characterized by the fixed-point problem:

$$\hat{h} = \arg\max_{h \geq \bar{h}} \pi(h, \hat{h}), \tag{B1}$$

where $\pi(h, \hat{h})$ is the monopolistic investor's profit function given by equation (8). This off-equilibrium belief is defined in the sense of subgame perfection. Intuitively, when acquiring information, the investor may first decide whether to conduct a site visit (i.e., whether $h \geq \bar{h}$ or $h \leq \bar{h}$) and then decide on the specific information acquisition details in each regime (i.e., the specific values of h). Facing $m = SV$, the market maker infers that the investor must have conducted a site visit (recall that the investor cannot misreport). Equation (B1) then constitutes an equilibrium in the subgame starting from that the investor has decided to conduct a site visit and thus can only choose $h \geq \bar{h}$: (i) the investor's precision choice maximizes her expected profit given the market maker's belief; and (ii) the market maker's belief is consistent with the investor's optimal precision choice.

We now follow the logic of equation (B1) to define off-equilibrium beliefs. We first introduce some useful notations. Recall that, under the site-visit application and combining voluntary and mandatory disclosures, messages can only take three possible values: SV, NV, and $\varnothing$. These messages constrain the possibilities of h_j. Let us introduce the following set function $B(m_j)$ to represent these constraints:

$$B\left(m_j\right) = \begin{cases} [\bar{h}, \infty), & \text{if } m_j = SV, \\ [0, \bar{h}], & \text{if } m_j = NV, \\ [0, \infty), & \text{if } m_j = \varnothing. \end{cases} \tag{B2}$$

Equipped with the set function $B(\cdot)$, we can extend equation (B1) and define off-equilibrium beliefs $\hat{h}_j(m_j)$ as follows:

$$\hat{h}_j\left(m_j\right) = \arg\max_{h_j \in B(m_j)} \pi_j(h_j, \hat{h}_j(m_j); h^*), \tag{B3}$$

where $\pi_j(h_j, \hat{h}_j; h^*)$ is equation (14) which replaces λ and α_{-j} with expressions (15) and (16), respectively. Note that equation (B3) is also satisfied in the message on the equilibrium path because the on-equilibrium consistency is a stronger notion.[23] We directly incorporate the belief specification (B3) in the following equilibrium definition.

Definition B1 (Equilibrium with the voluntary disclosure of site visits)**.** *An equilibrium in an economy with the voluntary disclosure of site visits consists of investors' information precision levels* $\{h_j^*\}_{j=1}^J$*, investor* j*'s message function* $m_j(h_j) : [0, +\infty) \to \{\varnothing, SV\}, \forall j$*, other agents' beliefs about investor* j*'s precision,* $\hat{h}_j(m_j) : \{\varnothing, SV\} \to [0, \infty), \forall j$*, investors' trading strategies* $\{\alpha_j(m_1, ..., m_J) \cdot \tilde{y}_j\}_{j=1}^J$*, and the market maker's pricing rule* $\lambda(m_1, ..., m_J) \cdot \tilde{\omega}$*, such that:*

(1) *On date 0:*

(1.1) *Precision levels* $\{h_j^*\}_{j=1}^J$ *form a Nash equilibrium, as in Part (1) of Definition 1. That is,* $h_j^* = \arg\max_{h_j} \pi_j\left(h_j, \hat{h}_j(m_j(h_j)); \{h_k^*\}_{k\neq j}\right), \forall j$.

(1.2) *Given acquired precision* h_j*, if* $h_j < \bar{h}$*, then the investor reports* $m_j(h_j) = \varnothing$*; if* $h_j \geq \bar{h}$*, then the investor's reported precision* $m_j(h_j) \in \{\varnothing, SV\}$ *maximizes her expected profits evaluated at the date-1 equilibrium, that is,* $m_j(h_j) = \arg\max_{m_j \in \{\varnothing, SV\}} \pi_j\left(h_j, \hat{h}_j(m_j); \{h_k^*\}_{k\neq j}\right)$.

(2) *On date 1, investors' trading strategies and the market maker's pricing rule form a trading equilibrium that is characterized by Part (2) of Definition 1, and agents use the belief function* $\hat{h}_j(m_j)$ *to update their beliefs* $\hat{h}_j$ *about investor* j*'s information precision.*

(3) *The belief functions satisfy the following consistency properties:*

(3.1) *On the equilibrium path:* $\hat{h}_j\left(m_j\left(h_j^*\right)\right) = h_j^*, \forall j$.

(3.2) *Off the equilibrium path:* $\hat{h}_j(m_j)$ *is given by equation (B3) for* $m_j \neq m_j\left(h_j^*\right), \forall j$.

Parts (1.1) and (2) of Definition B1 correspond respectively to Parts (1) and (2) of Definition 1, the equilibrium definition with exogenously given message functions. Part (1.2) of Definition

[23] Formally, on the equilibrium path, equation (B3) is equivalent to $\pi_j(h^*, h^*; h^*) \geq \pi_j(h_j, h^*; h^*)$ for all $h_j \in B(m_j(h^*))$. The optimality of equilibrium precision implies that $\pi_j(h^*, h^*; h^*) \geq \pi_j(h_j, \hat{h}_j(m_j(h_j)); h^*)$ for all $h_j \geq 0$. The latter condition is stronger because the optimal precision choice has considered all possible deviations of h_j without constraining it in $B(m_j(h^*))$ (i.e., "$h_j \geq 0$" versus "$h_j \in B(m_j(h^*))$"). In addition, under voluntary disclosure, the investor optimally chooses messages to affect other agents' beliefs and further improve her expected profits (i.e., $\pi_j(h_j, \hat{h}_j(m_j(h_j)); h^*) \geq \pi_j(h_j, h^*; h^*)$).

B1 describes how investors optimally report their information-precision levels, which, in turn, endogenizes the message functions. Part (3) specifies beliefs both on and off the equilibrium path. We can define an equilibrium with mandatory disclosure of site visits that is similar to Definition B1; in this case, however, the message function is exogenously given by $m_j(h_j) = NV$ if $h_j \leq \bar{h}$ and $m_j(h_j) = SV$ if $h_j \geq \bar{h}$ and, thus, Part (1.2) of Definition B1 becomes irrelevant.

B2. Formal Results in the Oligopolistic Settings

Lemma B1 (Voluntary disclosure – Oligopolistic investors)**.** *In the oligopolistic-investor setting with the voluntary disclosure of site visits, the following holds:*

(1) *For sufficiently high c/σ_u, there exists a unique symmetric equilibrium in which (i) investors acquire information of precision h_s, where h_s is the unique solution to equation (12), (ii) investors conceal their information precision (i.e., $m_j(h_j) = \varnothing$), (iii) the other agents' beliefs about investor j's information precision is $\hat{h}_j(\varnothing) = h_s$ (on equilibrium), $\hat{h}_j(SV) = h_s$ (off equilibrium) when $h_s > \bar{h}$, and $\hat{h}_j(SV) = \bar{h}$ (off equilibrium) when $h_s < \bar{h}$, and (iv) the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with λ given by (15) and investors' trading rule is $\tilde{x}_j = \alpha\tilde{y}_j$ with α given by (16).*

(2) *For sufficiently low c/σ_u:*

(2.1) *If $\bar{h} \leq h_s$ or $\bar{h} \to \infty$, then there exists a unique symmetric equilibrium in which (i) investors acquire information of precision h_s, (ii) investors conceal their information precision (i.e., $m_j(h_j) = \varnothing$), (iii) the other agents' beliefs about investor j's information precision is $\hat{h}_j(\varnothing) = h_s$ (on equilibrium), $\hat{h}_j(SV) = h_s$ (off equilibrium) when $h_s > \bar{h}$, and $\hat{h}_j(SV) = \bar{h}$ (off equilibrium) when $\bar{h} \to \infty$, and (iv) the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with λ given by (15) and investors' trading rule is $\tilde{x}_j = \alpha\tilde{y}_j$ with α given by (16).*

(2.2) *If $h_s < \bar{h} \leq h_o$, where h_o is the unique solution to equation (13), then there exists a unique symmetric equilibrium in which (i) investors conduct site visits to acquire information of precision $\bar{h}$, (ii) investors disclose their conducted site visits (i.e., $m_j(h_j) = SV$ if $h_j \geq \bar{h}$*

and $m_j(h_j) = \varnothing$ if $h_j < \bar{h}$), (iii) the other agents' beliefs about investor j's information precision is $\hat{h}_j(SV) = \bar{h}$ (on equilibrium) and $\hat{h}_j(\varnothing) < h_s$ (off equilibrium), and (iv) the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with λ given by (15) and investors' trading rule is $\tilde{x}_j = \alpha\tilde{y}_j$ with α given by (16).

Lemma B2 (Mandatory disclosure – Oligopolistic investors)**.** *In the oligopolistic-investor setting with the mandatory disclosure of site visits, the following holds:*

(1) *When c/σ_u is sufficiently high,*

(1.1) *If $\bar{h} \to 0$, there exists a unique symmetric equilibrium in which (i) investors conduct site visits and acquire information of precision h_s, (ii) the belief function is $\hat{h}_j(SV) = h_s$ and $\hat{h}_j(NV) = \bar{h}$, and (iii) the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with λ given by (15) and investors' trading rule is $\tilde{x}_j = \alpha\tilde{y}_j$ with α given by (16).*

(1.2) *If $h_o \leq \bar{h} \leq h_s$, there exists a unique symmetric equilibrium in which (i) investors do not conduct site visits and acquire information of precision $\bar{h}$, (ii) the belief function is $\hat{h}_j(NV) = \bar{h}$ and $\hat{h}_j(SV) \geq \bar{h}$, and (iii) the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with λ given by (15) and investors' trading rule is $\tilde{x}_j = \alpha\tilde{y}_j$ with α given by (16).*

(1.3) *If $\bar{h} > h_s$, there exists a unique symmetric equilibrium in which (i) investors do not conduct site visits and acquire information of precision h_s, (ii) the belief function is $\hat{h}_j(NV) = h_s$ and $\hat{h}_j(SV) = \bar{h}$, and (iii) the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with λ given by (15) and investors' trading rule is $\tilde{x}_j = \alpha\tilde{y}_j$ with α given by (16).*

(2) *When c/σ_u is sufficiently low,*

(2.1) *If $\bar{h} \leq h_s$, there exists a unique symmetric equilibrium in which (i) investors conduct site visits and acquire information of precision h_s, (ii) the belief function is $\hat{h}_j(SV) = h_s$ and $\hat{h}_j(NV) = \bar{h}$, and (iii) the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with λ given by (15) and investors' trading rule is $\tilde{x}_j = \alpha\tilde{y}_j$ with α given by (16).*

(2.2) *If $h_s < \bar{h} \le h_o$, there exists a unique symmetric equilibrium in which (i) investors conduct site visits and acquire information of precision $\bar{h}$, (ii) the belief function is $\hat{h}_j(SV) = \bar{h}$ and $\hat{h}_j(NV) \le \bar{h}$, and (iii) the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with λ given by (15) and investors' trading rule is $\tilde{x}_j = \alpha\tilde{y}_j$ with α given by (16).*

(2.3) *If $\bar{h} \to \infty$, there exists a unique symmetric equilibrium in which (i) investors do not conduct site visits and acquire information of precision h_s, (ii) the belief function is $\hat{h}_j(NV) = h_s$ and $\hat{h}_j(SV) = \bar{h}$, and (iii) the market maker's pricing rule is $\tilde{p} = \lambda\tilde{\omega}$ with λ given by (15) and investors' trading rule is $\tilde{x}_j = \alpha\tilde{y}_j$ with α given by (16).*

Proof of Lemmas B1 and B2

Following the same derivation in the proof of Lemma 2, we can compute that, given her information precision h_j, other agents' beliefs $\hat{h}_j$ about her information precision, and other investors' information precision $h_{-j} = h^*$, investor j's date-0 expected net trading profit $\pi_j(h_j, \hat{h}_j; h^*)$ is given by (A5). We follow the same procedures as we followed in the proofs of Lemmas 4 and 5 and first present an auxiliary lemma that helps the subsequent proof.

Lemma B3. *The following facts hold under oligopoly:*

(1) $\lim_{c/\sigma_u \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} < 0$. *That is, when $c/\sigma_u \to \infty$, regardless of h_j and h^*, investor j wants other agents' beliefs about her information precision to be the lowest possible.*

(2) $\lim_{c/\sigma_u \to 0} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} > 0$. *That is, when $c/\sigma_u \to 0$, regardless of h_j and h^*, investor j wants the other agents' beliefs about her information precision to be the highest possible.*

(3) $\frac{\partial \pi_j(h_j, h_j; h_o)}{\partial h_j}|_{h_j = h_o} = 0$ *and* $\frac{\partial^2 \pi_j(h_j, h_j; h_o)}{\partial h_j^2} < 0$. *That is, $\pi_j(h_j, h_j; h_o)$ is single peaked at $h_j = h_o$.*

(4) $\frac{\partial \pi_j(h_j, h_s; h_s)}{\partial h_j}|_{h_j = h_s} = 0$ *and* $\frac{\partial^2 \pi_j(h_j, h_s; h_s)}{\partial h_j^2} < 0$. *That is, $\pi_j(h_j, h_s; h_s)$ is single peaked at $h_j = h_s$.*

(5) *Given that $h^* \le h_s$, $\pi_j(h_j, h^*; h^*)$ increases in h_j when $h_j \le h_s$.*

(6) *Given that $h^* \ge h_o$, $\pi_j(h_j, h_j; h^*)$ decreases in h_j when $h_j > h^*$.*

(7) *Given that* $h^* \leq h_o$, $\pi_j(h_j, h_j; h^*)$ *increases in* h_j *when* $h_j < h^*$.

(8) *Given that* $c/\sigma_u \to \infty$ *and* $h^* \geq h_o$, $\pi_j(h_j, h_j; h^*)$ *decreases in* h_j *when* $h_j \geq h_o$.

(9) *Given that* $c/\sigma_u \to 0$ *and* $h^* \leq h_o$, $\pi_j(h_j, h_j; h^*)$ *increases in* h_j *when* $h_j \leq h_o$.

Proof. Parts (3) and (4) are straightforward and we skip their proofs here. For Parts (1) and (2), taking derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ as given by (A5) with respect to $\hat{h}_j$, and imposing $h^* \to 0$ we obtain that $\lim_{h^* \to 0} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} = -\frac{h_j(1+\hat{h}_j)\sigma_u}{4(1+h_j)(\hat{h}_j(1+\hat{h}_j))^{3/2}} < 0$. Furthermore, based on equation (A5), it is straightforward that regardless of other agents' belief $\hat{h}_j$ and other investors' information precision h_{-j}, as $c/\sigma_u \to \infty$, investor j's optimal information acquisition $h_j \to 0$, so in equilibrium $h^* \to 0$. That is, $\lim_{c/\sigma_u \to \infty} h^* = 0$. Taken together, $\lim_{c/\sigma_u \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} < 0$. Similarly, taking derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ with respect to $\hat{h}_j$ and imposing $h^* \to \infty$ yields the following:

$$\lim_{h^* \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} = \frac{h_j(1+\hat{h}_j)\sigma_u \cdot \Psi_1(\hat{h}_j)}{2(1+h_j)(\hat{h}_j + \hat{h}_j J + 2J)^2 \left(\hat{h}_j^2 J + \hat{h}_j(4J-3) + 4(J-1)\right)^{3/2}} > 0.$$

The positive sign holds because $\Psi_1(\hat{h}_j) = \hat{h}_j^2(4J^2 - 3J - 3) + \hat{h}_j(16J^2 - 21J + 1) + (16J^2 - 26J + 8) > 0$, where the inequality follows because the discriminant $(16J^2 - 21J + 1)^2 - 4(4J^2 - 3J - 3)(16J^2 - 26J + 8) < 0$ and when $J \geq 2$, $4J^2 - 3J - 3 > 0$. That is, $\Psi_1(\hat{h}_j)$ opens upward and does not interact with the x-axis. Coupled with the fact that $\lim_{c/\sigma_u \to 0} h^* = \infty$, we obtain that $\lim_{c/\sigma_u \to 0} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} > 0$.

Part (5) holds because $\frac{\partial \pi_j(h_j, h^*; h^*)}{\partial h_j} = -c + \frac{\sigma_u(1+h^*)^2}{(1+h_j)^2\sqrt{h^*(1+h^*)J}(2+h^*+h^*J)} \geq -c + \frac{\sigma_u}{(2+h_s+h_sJ)\sqrt{h_s(1+h_s)J}} = 0$, where the inequality holds because $\frac{\partial \pi_j(h_j, h^*; h^*)}{\partial h_j}$ decreases in both h_j and h^* and $h_j, h^* \leq h_s$, and the last equality follows from the definition of h_s.

Part (6) holds because $\frac{\partial \pi_j(h_j, h_j; h^*)}{\partial h_j} < \frac{\partial \pi_j(h_j, h_j; h^*)}{\partial h_j}|_{h_j = h^*} \leq \frac{\partial \pi_j(h_j, h_j; h_o)}{\partial h_j}|_{h_j = h_o} = 0$, where the first inequality follows because $\frac{\partial^2 \pi_j(h_j, h_j; h^*)}{\partial h_j^2} < 0$ and $h_j > h^*$, the second inequality follows because $\frac{\partial}{\partial h^*}\left(\frac{\partial \pi_j(h_j, h_j; h^*)}{\partial h_j}|_{h_j = h^*}\right) < 0$ and $h^* \geq h_o$, and the last equality follows from the definition of h_o. Similarly, Part (7) follows because $\frac{\partial \pi_j(h_j, h_j; h^*)}{\partial h_j} > \frac{\partial \pi_j(h_j, h_j; h^*)}{\partial h_j}|_{h_j = h^*} \geq \frac{\partial \pi_j(h_j, h_j; h_o)}{\partial h_j}|_{h_j = h_o} = 0$ and $h_j < h^* \leq h_o$.

Part (8) holds because $\frac{\partial \pi_j(h_j,h_j;h^*)}{\partial h_j} \leq \frac{\partial \pi_j(h_j,h_j;h_o)}{\partial h_j} \leq \frac{\partial \pi_j(h_j,h_j;h_o)}{\partial h_j}|_{h_j=h_o} = 0$, where the second inequality follows because $\frac{\partial^2 \pi_j(h_j,h_j;h_o)}{\partial h_j^2} < 0$ and $h_j \geq h_o$. The first inequality follows because $h^* \geq h_o$ and when $c/\sigma_u \to \infty$, $\frac{\partial}{\partial h^*}\left(\frac{\partial \pi_j(h_j,h_j;h^*)}{\partial h_j}\right) < 0$. Specifically,

$$\frac{\partial}{\partial h^*}\left(\frac{\partial \pi_j(h_j,h_j;h^*)}{\partial h_j}\right) = \frac{(h^*+2)\,\sigma_u \times \Psi_2(h_j,h^*)}{\left(\begin{array}{c} 4\left(h_j\left(h^*(J+1)+4\right)+2h^*J+4\right)^3 \\ \times \left(\begin{array}{c} h_j^2\left(h^*\left(h^*J+J+3\right)+4\right) \\ +h_j\left(h^{*2}(4J-3)+4h^*J+4\right) \\ +4h^*\left(h^*+1\right)(J-1) \end{array}\right)^{5/2} \end{array}\right)}, \tag{B4}$$

where $\Psi_2(h_j,h^*)$ is a function of h_j and h^*. When $c/\sigma_u \to \infty$ so that $h^* \to 0$ and $h_j \to 0$, $\frac{\partial}{\partial h^*}\left(\frac{\partial \pi_j(h_j,h_j;h^*)}{\partial h_j}\right) \to \frac{\sigma_u \times 512(J-1)(h_j-2(J-1)h^*)}{4096(h_j+(J-1)h_s)} < -\frac{(h^*+2)\sigma_u \times 512(J-1)(2J-3)h^*}{8192(h_j+(J-1)h_s)} < 0$, where the first inequality follows because $h_j < h^*$. Similarly, Part (9) follows because $\frac{\partial \pi_j(h_j,h_j;h^*)}{\partial h_j} \geq \frac{\partial \pi_j(h_j,h_j;h_o)}{\partial h_j} \geq \frac{\partial \pi_j(h_j,h_j;h_o)}{\partial h_j}|_{h_j=h_o} = 0$, where the second inequality follows because $\frac{\partial^2 \pi_j(h_j,h_j;h^*)}{\partial h_j^2} < 0$ and $h_j \leq h_o$. The first inequality follows because $h^* \leq h_o$ and when $c/\sigma_u \to 0$, $\frac{\partial}{\partial h^*}\left(\frac{\partial \pi_j(h_j,h_j;h^*)}{\partial h_j}\right) < 0$; specifically, $\frac{\partial}{\partial h^*}\left(\frac{\partial \pi_j(h_j,h_j;h^*)}{\partial h_j}\right)$ is given by (B4) and when $c/\sigma_u \to 0$ so that $h^* \to \infty$ and $h_j \to \infty$, we have $\frac{\partial}{\partial h^*}\left(\frac{\partial \pi_j(h_j,h_j;h^*)}{\partial h_j}\right) \to -\frac{\sigma_u(27+33J-37J^2-65J^3+42J^4)}{4J^{5/2}(J+1)^3 h_j^2 h^{*2}} < 0$. □

Scenario 1: Sufficiently High c/σ_u

Mandatory Disclosure (Part (1) of Lemma B2)

Case A. $\bar{h} \leq h_s$ and $h^* \leq \bar{h}$. The condition $h^* \leq \bar{h}$ suggests that the candidate symmetric equilibrium involves no site visits. Given $h_{-j} = h^*$, since $h_j = h^*$ maximizes investor j's profit across all precision levels, it must also maximize her profit subject to $h_j \leq \bar{h}$. That is, $h^* = \arg\max_{h_j \leq \bar{h}} \pi_j(h_j, \hat{h}_j; h^*)$, where $\pi_j(h_j, \hat{h}_j; h^*)$ is given by equation (A5). The Lagrangian function is

$$\mathcal{L}_3 = \pi_j(h_j, \hat{h}_j; h^*) + \mu(\bar{h} - h_j), \tag{B5}$$

where $\mu \geq 0$ is the Lagrangian multiplier. Taking the derivative with respect to h_j and imposing symmetric equilibrium and consistent beliefs ($h_j = \hat{h}_j = h^*$) yield the following:

$$\frac{\sigma_u}{\sqrt{h^*(1+h^*)J}(2+h^*+h^*J)} = c + \mu. \tag{B6}$$

The complementary slackness conditions are $\mu \geq 0$, $h^* \leq \bar{h}$, and $\mu(\bar{h} - h^*) = 0$.

If $\mu = 0$, the only solution to (B6) is $h^* = h_s$. Given the constraint $h_j \leq \bar{h} \leq h_s$, this equilibrium can be sustained only when $\bar{h} = h_s$, which can be combined in the following case of $\mu > 0$.

If $\mu > 0$, the constraint must be binding and, thus, $h^* = \bar{h}$. Given that $\frac{\sigma_u}{\sqrt{h_s(1+h_s)J}(2+h_s+h_sJ)} = c$, $\bar{h} \leq h_s$ and $\frac{\sigma_u}{\sqrt{h(1+h)J}(2+h+hJ)}$ decreases in h, we can always find a $\mu \geq 0$, such that $\bar{h}$ solves equation (B6). In the candidate equilibrium, investor j makes profit $\pi_j(\bar{h}, \bar{h}; \bar{h})$. Will investor j deviate to $h_j \geq \bar{h}$ (i.e., site visits)? When $h_j \geq \bar{h}$, other agents update their beliefs to $\hat{h}_j(SV) \geq \bar{h}$ according to equation (B3). Thus, the candidate equilibrium $h^* = \bar{h}$ can be sustained when $\pi_j(\bar{h}, \bar{h}; \bar{h}) \geq \pi_j(\hat{h}_j(SV), \hat{h}_j(SV); \bar{h})$. Although it is difficult to analytically characterize when this inequality holds, we can prove the following statements: (i) When $h_o \leq \bar{h} \leq h_s$, $\bar{h}$ is an equilibrium due to Part (6) of Lemma B3; and (ii) when $\bar{h} \to 0$, $\bar{h}$ cannot be sustained because $\pi_j(\bar{h}, \bar{h}; \bar{h}) \to 0$ and the deviation must yield positive profit for investor j.

Result B2.1. *(1) When $h_o \leq \bar{h} \leq h_s$, there exists a symmetric equilibrium in which investors do not conduct site visits and acquire information of precision $\bar{h}$ and in which the belief is $\hat{h}(NV) = \bar{h}$ (on equilibrium) and $\hat{h}(SV) \geq \bar{h}$ (off equilibrium). (2) When $\bar{h} \to 0$, there does not exist a symmetric equilibrium in which investors do not conduct site visits.*

Case B. $\bar{h} \leq h_s$ and $h^* \geq \bar{h}$. The condition $h^* \geq \bar{h}$ means that investors conduct site visits. Given $h_{-j} = h^*$, as $h_j = h^*$ maximizes investor j's profit across all precision levels, it must also maximize her profit subject to $h_j \geq \bar{h}$. That is, $h^* = \arg\max_{h_j \geq \bar{h}} \pi_j(h_j, \hat{h}_j; h^*)$, where $\pi_j(h_j, \hat{h}_j; h^*)$ is given by equation (A5). The Lagrangian function is

$$\mathcal{L}_4 = \pi_j(h_j, \hat{h}_j; h^*) + \mu(h_j - \bar{h}), \tag{B7}$$

where $\mu \geq 0$ is the Lagrangian multiplier. Taking the derivative with respect to h_j and imposing symmetric equilibrium and consistent belief ($h_j = \hat{h}_j = h^*$) yield the following:

$$\frac{\sigma_u}{\sqrt{h^*(1+h^*)J}(2+h^*+h^*J)} = c - \mu. \tag{B8}$$

The complementary slackness conditions are $\mu \geq 0$, $h^* \leq \bar{h}$, and $\mu(h^* - \bar{h}) = 0$.

If $\mu > 0$, the constraint is binding: $h^* = \bar{h}$. However, equation (B8) suggests that $c - \mu = \frac{\sigma_u}{\sqrt{\bar{h}(1+\bar{h})J}(2+\bar{h}+\bar{h}J)} \geq \frac{\sigma_u}{\sqrt{h_s(1+h_s)J}(2+h_s+h_sJ)} = c$, which holds only when $\mu = 0$, contradicting that $\mu > 0$. So, $\bar{h}$ cannot be a symmetric equilibrium.

If $\mu = 0$, then the only solution to equation (B8) is $h^* = h_s$ and the resulting profit is $\pi_j(h_s, h_s; h_s)$. Will investor j deviate to $h_j \leq \bar{h}$ (i.e., no site visits)? If investor j deviates, equation (B3) dictates that other agents revise their beliefs to $\hat{h}_j(NV) = \bar{h}$. Thus, $h^* = h_s$ can be sustained when $\pi_j(h_s, h_s; h_s) \geq \pi_j(\hat{h}_j(NV), \hat{h}_j(NV); h_s) = \pi_j(\bar{h}, \bar{h}; h_s)$. As in Case A, we cannot pin down the range of $\bar{h}$ in general. However, we can show the following: (i) When $h_o \leq \bar{h} \leq h_s$, by the fact that $h_s > h_o$ (see Proposition 2) and Part (8) of Lemma B3, $\pi_j(\bar{h}, \bar{h}; h_s) \geq \pi_j(h_s, h_s; h_s)$, so h_s cannot be sustained in equilibrium; and (ii) when $\bar{h} \to 0$, by deviating to $h_j < \bar{h}$ investor j makes zero profit whereas $\pi_j(h_s, h_s; h_s) > 0$, so h_s can be sustained in equilibrium.

Result B2.2. *(1) When $h_o \leq \bar{h} \leq h_s$, there does not exist a symmetric equilibrium in which investors conduct site visits. (2) When $\bar{h} \to 0$, there exists a symmetric equilibrium in which investors conduct site visits and acquire information of precision h_s and in which the belief is $\hat{h}(SV) = h_s$ (on equilibrium) and $\hat{h}(NV) = \bar{h}$ (off equilibrium).*

Case C. $\bar{h} > h_s$ and $h^* \leq \bar{h}$. In this case, h^* solves the same constrained maximization problem as in (B6). If $\mu > 0$, the constraint is binding and the solution is $h^* = \bar{h}$. However, equation (B6) suggests that $c + \mu = \frac{\sigma_u}{\sqrt{J(\bar{h})(1+\bar{h})}(2+\bar{h}+J(\bar{h}))} < \frac{\sigma_u}{\sqrt{Jh_s(1+h_s)}(2+h_s+h_sJ)} = c$, contradicting that $\mu > 0$. Thus, $\bar{h}$ cannot be sustained in equilibrium.

If $\mu = 0$, the unique solution to (B6) is $h^* = h_s$. Will investor j deviate to $h_j \geq \bar{h}$ (i.e., site visits)? Upon deviation, equation (B3) dictates that other agents revise beliefs to $\hat{h}_j(SV) = \bar{h}$. So, after deviation, investor j makes profits $\pi_j(\bar{h}, \bar{h}; h_s) < \pi_j(\bar{h}, h_s; h_s) < \pi_j(h_s, h_s; h_s)$, where the two inequalities follow Parts (1) and (4) of Lemma B3, respectively. So, h_s can be sustained as a symmetric equilibrium.

Result B2.3. *When $\bar{h} > h_s$, there exists a symmetric equilibrium in which investors do not conduct site visits and acquire information of precision h_s and in which the belief is $\hat{h}(NV) = h_s$ (on equilibrium) and $\hat{h}(SV) = \bar{h}$ (off equilibrium).*

Case D. $\bar{h} > h_s$ and $h^* \geq \bar{h}$. In this case, h^* solves the same problem as that in (B8). If $\mu = 0$, the unique solution is $h^* = h_s$, which contradicts that $h_j \geq \bar{h} > h_s$, so it cannot be sustained.

If $\mu > 0$, the unique solution to (B8) is $h^* = \bar{h}$. Will investor j deviate to $h_j \leq \bar{h}$ (i.e., no site visits)? Yes. By deviating to $h_o < h_j < \bar{h}$, other agents revise their beliefs to $\hat{h}_j(NV) \leq \bar{h}$ according to equation (B3), and investor j makes profits $\pi_j(\hat{h}_j(NV), \hat{h}_j(NV); \bar{h}) \geq \pi(\bar{h}, \bar{h}; \bar{h})$, where the inequality follows Part (8) of Lemma B3.

Result B2.4. *When $\bar{h} > h_s$, there does not exist a symmetric equilibrium in which investors conduct site visits.*

In sum, Results B2.1 through B2.2 deliver Parts (1.1) and (1.2) of Lemma B2, and Results B2.3 through B2.4 deliver Part (1.3) of Lemma B2.

Voluntary Disclosure (Part (1) of Lemma B1)

Case A'. $\bar{h} \leq h_s$ and $h^* \leq \bar{h}$. As discussed in Case A, the candidate equilibrium is $h^* = \bar{h}$ (i.e., no site visits), and, in equilibrium, investors are unable to make disclosure since they are not conducting site visits. Will investor j deviate to $h_j \geq \bar{h}$ (i.e., site visits)? Yes. If the investor deviates to $h_j \in (\bar{h}, h_s]$ and remains silent, other agents stick to the belief $\hat{h}(\varnothing) = \bar{h}$ and she makes profit $\pi_j(h_j, \bar{h}; \bar{h}) > \pi_j(\bar{h}, \bar{h}; \bar{h})$, where the inequality follows Part (5) of Lemma B3. So, $\bar{h}$ cannot be sustained as a symmetric equilibrium.

Result B1.1. *When $\bar{h} \leq h_s$, there does not exist a symmetric equilibrium in which investors do not conduct site visits.*

Case B'. $\bar{h} \le h_s$ and $h^* \ge \bar{h}$. As discussed in Case B, the candidate equilibrium is $h^* = h_s$ and investors are indifferent about disclosure. Given the belief $\hat{h}(SV) = \hat{h}(\varnothing) = h_s$, the candidate equilibrium can always be supported (i.e., $\pi_j(h_s, h_s; h_s) \ge \pi_j(h_j, h_s; h_s)$) by Part (4) of Lemma B3. Again, we assume that investors do not disclose since non-disclosure is robust against the introduction of an arbitrarily small disclosure cost.

Result B1.2. *When $\bar{h} \le h_s$, there exists a symmetric equilibrium in which investors conduct site visits and acquire information of precision h_s. The investors do not disclose: $m_j(h_j) = \varnothing$. And other agents' beliefs about investor j's information precision is $\hat{h}_j(\varnothing) = h_s$ (on equilibrium) and $\hat{h}_j(SV) = h_s$ (off equilibrium).*

Case C'. $\bar{h} > h_s$ and $h^* < \bar{h}$. As discussed in Case C, the candidate equilibrium is $h^* = h_s$ (i.e., no site visits) and investors are unable to disclose. Will investor j deviate to $h_j \ge \bar{h}$ and disclose? (Note that if the deviant investor remains silent, other agents' belief about her information precision is h_s and by Part (4) of Lemma B3 the deviation is not more profitable). The answer is no. After deviation, according to equation (B3), other agents revise their beliefs to $\hat{h}_j(SV) = \bar{h}$. And investor j's resulting profit is $\pi_j(\bar{h}, \bar{h}; h_s) < \pi_j(\bar{h}, h_s; h_s) \le \pi_j(h_s, h_s; h_s)$, where the first inequality follows $\bar{h} > h_s$ and Part (1) of Lemma B3 and the second follows Part (4) of Lemma B3. Thus, h_s can always be sustained.

Result B1.3. *When $\bar{h} > h_s$, there exists a symmetric equilibrium in which investors do not conduct site visits and acquire information of precision h_s. The investors do not disclose: $m_j(h_j) = \varnothing$. And other agents' beliefs about investor j's information precision is $\hat{h}_j(\varnothing) = h_s$ (on equilibrium) and $\hat{h}_j(SV) = \bar{h}$ (off equilibrium).*

Case D'. $\bar{h} > h_s$ and $h^* \ge \bar{h}$. As discussed in Case D, the only candidate symmetric equilibrium is $h^* = \bar{h}$ (i.e., site visits). Per Part (1) of Lemma B3, investor j always wants to keep silent since $\hat{h}_j(\varnothing) = h_s < \bar{h} \le \hat{h}_j(SV)$. However, given $\hat{h}_j = h_s$, the optimal precision level chosen by investors is h_s (no site visits). Thus, investors always want to deviate from $\bar{h}$ (i.e., site visits) to h_s (i.e., no site visits), which implies that $\bar{h}$ cannot be sustained as a symmetric equilibrium.

Result B1.4. *When $\bar{h} > h_s$, there does not exist a symmetric equilibrium in which investors conduct site visits.*

In sum, Results B1.1 through B1.4 deliver Part (1) of Lemma B1.

Scenario 2: Sufficiently Low c/σ_u

Mandatory Disclosure (Part (2) of Lemma B2)

Case I. $\bar{h} \le h_s$ and $h^* \le \bar{h}$. As discussed in Case A, the candidate symmetric equilibrium is $h^* = \bar{h}$ (i.e., no visits) and, in equilibrium, investor j makes profit $\pi_j(\bar{h}, \bar{h}; \bar{h})$. However, investor j always has incentives to deviate to $h_j \ge \bar{h}$ (i.e., site visit), and mandatory disclosure reveals the visit ($m_j = SV$). For instance, by deviating to $h_j \in [\bar{h}, h_o]$, other agents revise their beliefs to $\hat{h}_j(SV)$, which can be computed using equation (B3), and investor j's profit from deviation is $\pi_j(\hat{h}_j(SV), \hat{h}_j(SV); \bar{h}) \ge \pi_j(\bar{h}, \bar{h}; \bar{h})$, where the inequality follows from $\hat{h}_j(SV) \ge \bar{h}$ and Part (9) of Lemma B3. So, $\bar{h}$ cannot be sustained as a symmetric equilibrium.

Result B2.5. *When $\bar{h} \le h_s$, there does not exist a symmetric equilibrium in which investors do not conduct site visits.*

Case II. $\bar{h} \le h_s$ and $h^* \ge \bar{h}$. As discussed in Case B, the candidate symmetric equilibrium is $h^* = h_s$ (i.e., site visits). Will investor j deviate to $h_j \le \bar{h}$ (i.e., no site visits)? No. After deviation, other agents revise their beliefs to $\hat{h}_j(NV) = \bar{h}$, as computed by equation (B3). Thus, investor j's profit from deviation is $\pi_j(\bar{h}, \bar{h}; h_s) \le \pi_j(h_s, h_s; h_s)$, where the inequality follows the fact $h_s < h_o$ (see Proposition 2) and Part (7) of Lemma B3. So, h_s can be sustained as a symmetric equilibrium.

Result B2.6. *When $\bar{h} \le h_s$, there exists a symmetric equilibrium in which investors conduct site visits and acquire information of precision h_s and in which the belief is $\hat{h}_j(SV) = h_s$ (on equilibrium) and $\hat{h}_j(NV) = \bar{h}$ (off equilibrium).*

Case III. $\bar{h} > h_s$ and $h^* \le \bar{h}$. As discussed in Case C, the candidate symmetric equilibrium is $h^* = h_s$ (i.e., no visits). Will investor j deviate to $h_j \ge \bar{h}$ (i.e., site visits)? After deviating, based on equation (B3), other agents revise beliefs to $\hat{h}_j(SV) = \bar{h}$ and, hence, investor j makes profit $\pi_j(\bar{h}, \bar{h}; h_s)$. To support the equilibrium, we need $\pi_j(h_s, h_s; h_s) \geq \pi_j(\bar{h}, \bar{h}; h_s)$. It is difficult to analytically characterize when this inequality holds, but we can show the following: (i) When $h_s < \bar{h} \leq h_o$, $\pi_j(\bar{h}, \bar{h}; h_s) > \pi_j(h_s, h_s; h_s)$ per Part (7) of Lemma B3, so h_s cannot be sustained in equilibrium; and, (ii) When $\bar{h} \to \infty$, h_s can be sustained in equilibrium; otherwise if investor j deviates to $h_j = \bar{h}$, she only makes negative profits: $\lim_{\bar{h}\to\infty} \pi_j(\bar{h}, \bar{h}; h_s) < 0$.

Result B2.7. *(1) When $h_s < \bar{h} \leq h_o$, there does not exist a symmetric equilibrium in which investors conduct site visits. (2) When $\bar{h} \to \infty$, there exists a symmetric equilibrium in which investors do not conduct site visits and acquire information of precision h_s and in which the belief is $\hat{h}_j(NV) = h_s$ (on equilibrium) and $\hat{h}_j(SV) = \bar{h}$ (off equilibrium).*

<u>Case IV. $\bar{h} > h_s$ and $h^* \geq \bar{h}$.</u> As discussed in Case D, the candidate symmetric equilibrium is $h^* = \bar{h}$ (i.e., site visits). Will investor j deviate to $h_j \leq \bar{h}$ (i.e., no site visits)? After deviation, other agents revise their beliefs to $\hat{h}_j(NV) \leq \bar{h}$ based on equation (B3). And the candidate equilibrium can be sustained when $\pi_j(\bar{h}, \bar{h}; \bar{h}) \geq \pi_j(\hat{h}_j(NV), \hat{h}_j(NV); \bar{h})$. As in Case III, we can show the following: (i) When $h_s < \bar{h} \leq h_o$, $\bar{h}$ can be sustained as an equilibrium due to Part (9) of Lemma B3; and (ii) when $\bar{h} \to \infty$, $\pi_j(\bar{h}, \bar{h}; \bar{h}) < 0$ so $\bar{h}$ cannot be sustained as a symmetric equilibrium.

Result B2.8. *(1) When $h_s < \bar{h} \leq h_o$, there exists a symmetric equilibrium in which investors conduct site visits and investors acquire information of precision $\bar{h}$ and in which the belief is $\hat{h}_j(SV) = \bar{h}$ (on equilibrium) and $\hat{h}_j(NV) \leq \bar{h}$ (off equilibrium). (2) When $\bar{h} \to \infty$, there does not exist a symmetric equilibrium in which investors conduct site visits.*

In sum, Results B2.5 and B2.6 deliver Part (2.1) of Lemma B2 and Results B2.7 and B2.8 deliver Parts (2.2) and (2.3) of Lemma B2.

Voluntary Disclosure (Part (2) of Lemma B1)

<u>Case I'. $\bar{h} \leq h_s$ and $h^* \leq \bar{h}$.</u> As discussed in Case I, the candidate symmetric equilibrium is $h^* = \bar{h}$ (i.e., no visits) and investors are unable to disclose. In this candidate equilibrium, investors' profit

is $\pi_j(\bar{h}, \bar{h}; \bar{h})$. Will investor j deviate to $h_j \geq \bar{h}$ (i.e., site visits)? If the investor remains silent, other agents stick to the belief $\hat{h}_j(\varnothing) = \bar{h}$. So, by Part (5) of Lemma B3, $\pi_j(h_j, \bar{h}; \bar{h}) \geq \pi_j(\bar{h}, \bar{h}; \bar{h})$ when $h_j \in [\bar{h}, h_s]$; that is, $\bar{h}$ cannot be sustained in equilibrium. If the investor discloses after deviation, according to equation (B3), other agents update beliefs to $\hat{h}_j(SV) \geq \bar{h}$. The resulting profit for investor j is $\pi_j(\hat{h}_j(SV), \hat{h}_j(SV); \bar{h}) \geq \pi_j(\bar{h}, \bar{h}; \bar{h})$, where the inequality follows from $\hat{h}_j(SV) \geq \bar{h}$ and Part (9) of Lemma B3. Again, $\bar{h}$ cannot be sustained as a symmetric equilibrium.

Result B1.5. *When $\bar{h} \leq h_s$, there does not exist a symmetric equilibrium in which investors do not conduct site visits.*

Case II'. $\bar{h} \leq h_s$ and $h^* \geq \bar{h}$. As discussed in Case II, the candidate symmetric equilibrium is $h^* = h_s$ (site visits), and the investors are indifferent about disclosure. The other agents' beliefs must be $\hat{h}_j(SV) = \hat{h}_j(NV) = h_s$. Given this belief, h_s can always be sustained as a symmetric equilibrium per Part (4) of Lemma B3. We, again, assume that the investors do not disclose since non-disclosure is robust against the introduction of an arbitrarily small disclosure cost.

Result B1.6. *When $\bar{h} \leq h_s$, there exists a symmetric equilibrium in which investors conduct site visits and acquire information of precision h_s. The investors do not disclose: $m_j(h_j) = \varnothing$. And other agents' belief about investor j's information precision is $\hat{h}_j(\varnothing) = h_s$ (on equilibrium) and $\hat{h}_j(SV) = h_s$ (off equilibrium).*

Case III'. $\bar{h} > h_s$ and $h^* \leq \bar{h}$. As discussed in Case III, the candidate symmetric equilibrium is $h^* = h_s$ (i.e., no site visits) and investors are unable to disclose. Investors make profit $\pi_j(h_s, h_s; h_s)$ accordingly. Will investor j deviate to $h_j \geq \bar{h}$? If she deviates but remains silent, other agents stick to the belief $\hat{h}_j(\varnothing) = h_s$. By Part (4) of Lemma B3, we know that $\pi_j(h_s, h_s; h_s) \geq \pi_j(h_j, h_s; h_s)$; that is, h_s can be sustained in equilibrium. If the investor j discloses, other agents revise their beliefs to $\hat{h}_j(SV) = \bar{h}$ according to equation (B3). Following the same discussion in Case III, we know that: (i) When $h_s < \bar{h} \leq h_o$, h_s cannot be sustained in equilibrium, and (ii) when $\bar{h} \to \infty$, h_s can be sustained in equilibrium.

Result B1.7. *(1) When $h_s < \bar{h} \leq h_o$, there does not exist a symmetric equilibrium in which investors do not conduct site visits. (2) When $\bar{h} \to \infty$, there exists a symmetric equilibrium in which investors do not conduct site visits and acquire information of precision h_s. Investors make no disclosure: $m_j(h_j) = \varnothing$. The belief is $\hat{h}_j(\varnothing) = h_s$ (on equilibrium) and $\hat{h}_j(SV) = \bar{h}$ (off equilibrium).*

Case IV'. $\bar{h} > h_s$ and $h^* \geq \bar{h}$. As discussed in Case IV, the candidate symmetric equilibrium is $h^* = \bar{h}$ (i.e., site visits). Per Part (2) of Lemma B3, investor j wants to disclose since $\hat{h}_j(SV) = \bar{h} > h_s > \hat{h}_j(\varnothing)$, where the last strict inequality holds because of the following. We note that based on equation (B3), we can compute the off-equilibrium belief $\hat{h}_j(\varnothing)$, which is the solution to the following specific equation: $\frac{\partial \pi_j(h_j, \hat{h}_j; \bar{h})}{\partial h_j}|_{h_j = \hat{h}_j} = 0$. Since $\frac{\partial}{\partial h_j}\left(\frac{\partial \pi_j(h_j, \hat{h}_j; \bar{h})}{\partial h_j}|_{h_j = \hat{h}_j}\right) < 0$ and $\frac{\partial}{\partial \bar{h}}\left(\frac{\partial \pi_j(h_j, \hat{h}_j; \bar{h})}{\partial h_j}|_{h_j = \hat{h}_j}\right) < 0$, via implicit function theorem we know that $\frac{\partial \hat{h}_j(\varnothing)}{\partial \bar{h}} < 0$. That is, the off-equilibrium belief $\hat{h}_j(\varnothing)$ decreases in $\bar{h}$. Together with $\frac{\partial \pi_j(h_j, \hat{h}_j; h_s)}{\partial h_j}|_{h_j = \hat{h}_j = h_s} = 0$ and $\bar{h} > h_s$, we obtain that $\hat{h}_j(\varnothing) < h_s < \bar{h}$. That is, if investor j does not disclose, other agents will revise down their beliefs about her signal precision; per Part (2) of Lemma B3, investor j must then disclose in the candidate equilibrium. Next, will investor j deviate to $h_j \leq \bar{h}$ (i.e., no site visits)? After deviation, investor j is unable to disclose, and she makes profit $\pi_j(\hat{h}_j(\varnothing), \hat{h}_j(\varnothing); \bar{h})$. So, the candidate equilibrium $\bar{h}$ can be sustained when $\pi_j(\bar{h}, \bar{h}; \bar{h}) \geq \pi_j(\hat{h}_j(\varnothing), \hat{h}_j(\varnothing); \bar{h})$. As in Case IV, we cannot analytically characterize when this inequality holds; however, we can show the followings: (i) When $h_s < \bar{h} \leq h_o$, $\bar{h}$ can be sustained in equilibrium due to Part (7) of Lemma B3 and, thus, $\bar{h}$ can be sustained in equilibrium; and (ii) when $\bar{h} \to \infty$, $\bar{h}$ cannot be a symmetric equilibrium, because $\pi_j(\bar{h}, \bar{h}; \bar{h}) < 0$ and deviations can ensure non-negative profits.

Result B1.8. *(1) When $h_s < \bar{h} \leq h_o$, there exists a symmetric equilibrium in which investors conduct site visits and acquire information of precision $\bar{h}$. Investors disclose if they have conducted site visits: $m_j(h_j) = SV$ if $h_j \geq \bar{h}$ and $m_j(h_j) = \varnothing$ if $h_j < \bar{h}$. The belief is $\hat{h}_j(SV) = \bar{h}$ (on equilibrium) and $\hat{h}_j(\varnothing) < h_s$ (off equilibrium). (2) When $\bar{h} \to \infty$, there does not exist a symmetric equilibrium in which investors conduct site visits.*

In sum, Results B1.5 through B1.8 deliver Part (2) of Lemma B1.

Supplementary Online Appendix

Secret and Overt Information Acquisition in Financial Markets

by Xiong and Yang

This online appendix contains the following two sections. First, in Section S1, we discuss in more detail how to connect our model in Section 3 to corporate site visits. Second, in Section S2, we consider an alternative setup to model the site-visits context in which the disclosure threshold is pinned down endogenously under voluntary disclosure. We show that all our major results continue to hold in the alternative model and the driving forces are the same as in the main text.

S1 Mapping the Model to Corporate Site Visits

In this section, we provide more details supporting the discussions in Section 3.2, where we map the application of corporate site visits to our model. Specifically, we describe in greater detail the "segment" representation of information acquisition and the message functions in the voluntary and mandatory disclosure settings. In particular, we depict two figures to better illustrate the discussions.

In practice, investors acquire fundamental information about a company through a variety of means. Some methods generate more precise information than others but can also incur higher costs. Figure S1 depicts a rough order of these different information-acquisition methods, each of which is represented by an increasing line segment that reveals its fixed and variable costs as well as its limits in collecting precise information. More specifically, the left end of the segment shows that, at a fixed cost, the method delivers a minimum level of information precision. An investor can expend more resources to increase the precision level but only to a certain limit, which is represented by the right end of the segment. In Figure S1, the cheapest information-acquisition method is exploring the company's website and press releases; this practice is essentially free, but the information an investor can obtain is severely limited. If she wants to learn more, she can expend more resources to explore the company's financial reports and competitors or attend conference calls and non-deal roadshows.

Site visits are more advanced than most information-acquisition methods. Visiting a firm's headquarters or facilities allows the investor to engage with company employees and directly observe operations, which inevitably generates more precise information. At the same time, this method is costlier than others, given that the investor must exert efforts to maintain a strong relationship with the firm they are visiting and evaluating. We use two segments in Figure S1 to represent visits to headquarters and facilities.[1] If the investor visits a firm's headquarters, then she can collect a minimum level of information represented by a known constant $\bar{h}$. The visit's details determine the actual value of the investor's precision h_j, which, in turn, increases with the investor's efforts during the visit. If the investor spends a longer time at the firm, is more attentive in each conversation, visits more facilities, and/or engages more company employees, the information collected becomes more accurate. Of course, the investor can also combine site visits with other information-acquisition methods to pinpoint even more precise information; these combinations are represented by lines labeled as "combined" in Figure S1. For tractability, we employ a smooth and increasing cost function $C(h_j)$ to approximate these various information-acquisition methods. In our analysis, we continue to adopt a linear cost function $C(h_j) = c \cdot h_j$, which is plotted as the dashed line in Figure S1.

Next, we use Figure S1 to illustrate how we interpret the disclosure of site visits within our model. Suppose that an investor conducts a site visit and announces it to the public using hard evidence (e.g., the Castlefield disclosure report). The public then knows that the investor's information-precision level must correspond to "site visits" or "combined" in Figure S1. Thus, a message that discloses the investor's site visit, denoted by $m_j = SV$, can be interpreted as a lower bound of the investor's information precision (i.e., $h_j \geq \bar{h}$).

Figure S2 graphically illustrates the message function under voluntary and mandatory disclosures.

Under voluntary disclosure, if investor j conducts a site visit, then she can decide whether or not to disclose it. When investor j discloses (i.e., $m_j = SV$), other agents un-

[1]In our discussions, we implicitly assume that investors can conduct only one site visit because the literature shows that repeat visits of the same company by the same investor within one year are uncommon (e.g., Chen et al., 2022).

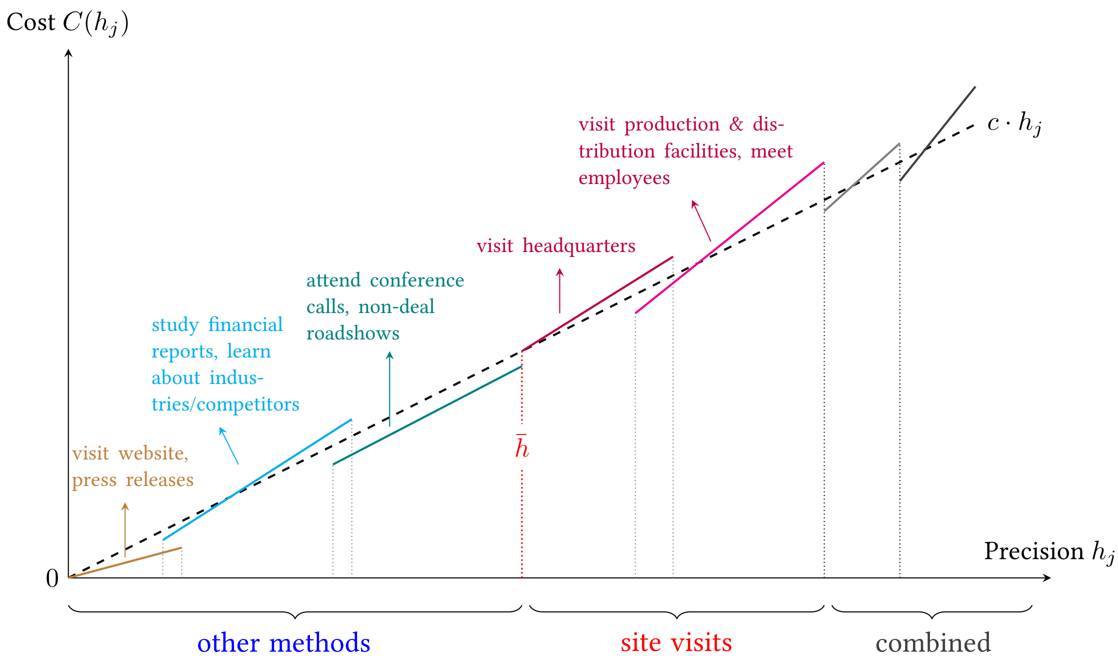

Figure S1: Fundamental analysis of a firm

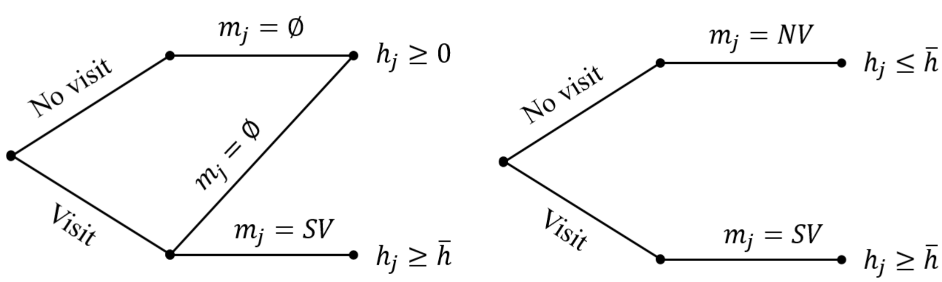

Figure S2: Voluntary disclosure vs. Mandatory disclosure

derstand that $h_j \geq \bar{h}$ and use this knowledge accordingly to form their beliefs about h_j. When investor j does not disclose (denoted by $m_j = \varnothing$), other agents do not directly observe any information about h_j (i.e., $h_j \geq 0$). Under voluntary disclosure, if the investor does not conduct a visit, she is incapable of credibly conveying this fact to the general public, which means that her message is $m_j = \varnothing$. As a result, if the investor announces nothing (i.e., $m_j = \varnothing$), the public cannot directly discern whether the investor did not conduct a visit or she did conduct a visit but chose not to release it.

Under mandatory disclosure, whether the investor conducted a site visit must be disclosed to the general public. If investor j conducts a site visit, her message is $m_j = SV$. Again, observing $m_j = SV$, other agents understand that $h_j \geq \bar{h}$. If investor j does not conduct a visit, this fact is mandatorily disclosed to the public, and the investor's message is denoted by $m_j = NV$. Observing $m_j = NV$, other agents understand that $h_j \leq \bar{h}$ and form their beliefs accordingly.

S2 An Alternative Model of Corporate Site Visits

In this section, we explore an alternative approach to modeling site visits, which does not involve an exogenous threshold $\bar{h}$. We will show that the results in the main text continue to hold in this alternative model and that the pricing and competition effects are still the driving forces. In Section S2.1, we explain how we model site visits under this alternative approach, and then, in Sections S2.2 and S2.3, we characterize the equilibrium and discuss model implications, respectively. All proofs are relegated to Section S2.4.

S2.1 Modeling Site Visits

We assume that all information is acquired via site visits. Suppose that each site visit by investor j, indexed by l, costs a fixed amount c_0 of resources and generates a signal $\tilde{v}+\tilde{z}_{j,l}$ about the company's fundamental value $\tilde{v}$, where $\tilde{z}_{j,l} \sim N(0, \sigma_z^2)$ (with $\sigma_z > 0$), and $\tilde{v}$, $\tilde{z}_{j,l}$, and $\tilde{z}_{j,l'}$ are mutually independent (for $l \neq l'$). Then, H_j visits generate a sufficient statistic $\tilde{v} + \tilde{e}_j$, where $\tilde{e}_j \equiv \frac{1}{H_j} \sum_{l=1}^{H_j} \tilde{z}_{j,l} \sim N(0, h_j^{-1})$ and $h_j \equiv H_j \sigma_z^{-2}$. Since σ_z is an exogenous constant, choosing precision h_j is the same as choosing the number H_j of site visits, and the information-acquisition cost is linear in h_j (i.e., $C(h_j) = c_0 \sigma_z^2 h_j$). By setting $c \equiv c_0 \sigma_z^2$ and noting that h_j can be approximated as a continuous variable when σ_z is large, we reproduce the information-acquisition technology described in our model.

Messages in this application are still implemented by disclosures. We next set forth the details for message specifications.

Voluntary Disclosure Suppose that a fund visits a company four times, say, in January, April, June, and October. Further assume that the fund discloses its January and April visits using hard evidence but does not mention anything about its visits in June and October. In this example, the general public knows that the fund has visited the company at least twice. In terms of notations, when investor j conducts H_j site visits, she can release a number M_j of the visits, where $M_j \leq H_j$. These M_j visits generate a sufficient statistic $\tilde{v} + \tilde{e}'_j$, where $\tilde{e}'_j \equiv \frac{1}{M_j} \sum_{l=1}^{M_j} \tilde{z}_{j,l} \sim N(0, m_j^{-1})$ and $m_j \equiv M_j \sigma_z^{-2}$. So, when investor j releases M_j visits, she informs the general public that she has acquired a private signal with precision of at least m_j, which, in turn, implies that the market maker and other investors believe that the precision of investor j's private signal must be at least m_j (i.e., $\hat{h}_j \geq m_j$). Note that the threshold m_j is endogenous since it is chosen by investor j.

To recap, under this alternative approach to modeling site visits, investor j chooses her information precision h_j by controlling the number of site visits she conducts. She then decides which site visits to disclose to the general public because she can only truthfully disclose those she has actually conducted. All agents understand this disclosure constraint; as such, when investor j reports m_j, other agents form a belief $\hat{h}_j \geq m_j$. All three variables $(h_j, m_j, \hat{h}_j)$ are endogenous in the voluntary disclosure setting.

Mandatory Disclosure To the extent that the implementation of regulations is effective, this mandatory disclosure setting degenerates to the overt benchmark economy analyzed in Section 2. Specifically, strict regulations require every one of an investor's information-acquisition activities to be reported, which, in turn, implies that the reported information precision is equal to the acquired precision (i.e., $m_j = h_j$). Hence, the other agents' beliefs are adjusted with the acquired precision (i.e., $\hat{h}_j = h_j$). Since we have analyzed the overt benchmark's equilibrium in the main text, we will not repeat it in this appendix.

S2.2 Voluntary Disclosure of Information Acquisition

This voluntary disclosure setting is a type of endogenous signaling game in that the message sent by an investor conveys information about her *endogenous* information-acquisition choices. Relative to Definition 1 for economies with exogenous message functions, the equilibrium in this voluntary disclosure setting requires specifying (1) how investors choose reported precision m_j given their acquired precision h_j (i.e., message function $m_j(h_j)$) and (2) how other agents form beliefs $\hat{h}_j$ about an investor's information precision upon observing her message m_j, particularly when receiving an unexpected (off-equilibrium) message (i.e., belief function $\hat{h}_j(m_j)$). As common in the literature of endogenous signaling games (In and Wright, 2018), we must deal with off-equilibrium beliefs and possible equilibrium multiplicity. We follow the literature (e.g., McAfee and Schwartz, 1994; Rey and Vergé, 2004) and propose a criterion that acceptable beliefs should satisfy in equilibrium,[2] and refer to these beliefs as "strictly increasing beliefs."

Definition S1 (Strictly increasing beliefs)**.** *Under strictly increasing beliefs, other agents' beliefs $\hat{h}_j$ about investor j's information precision strictly increases in investor j's reported precision m_j; that is, the belief function $\hat{h}_j(m_j)$ is strictly increasing in m_j.*

In this setting, the notion of strictly increasing beliefs is compelling since the reported precision is the lower bound of the actual acquired precision. When investor j presents evidence for her information precision being at least m_j (in the example mentioned in Section S2.1, the fund discloses that its January and April visits, so it at least conducted two site visits), other agents update their beliefs and believe that investor j's actual information acquisition is at least as high as the disclosure level m_j. If the investor's reported precision m_j increases (e.g., in addition to the January and April visits, the fund also reports the June visit), then other agents will naturally revise up their beliefs about the in-

[2]Existing criteria for endogenous signaling games are not readily applicable to our setting. For example, Hart and Tirole (1990) and O'Brien and Shaffer (1992) use passive beliefs, under which the agents do not revise their beliefs when observing a disclosure different from what they expect in the candidate equilibrium. In our setting, if investor j makes an evidence-backed disclosure that's higher than the equilibrium level ($m_j > h^*$), other agents must update their beliefs to at least the disclosure level and, so, cannot hold a passive belief.

vestor's information precision (i.e., the belief function $\hat{h}_j(m_j)$ is increasing). We directly incorporate this increasing belief function into our equilibrium Definition S2 below.

Definition S2 (Alternative setup: Equilibrium with an endogenous communication technology)**.** *An equilibrium in the voluntary disclosure economy consists of investors' information precision levels* $\left\{h_j^*\right\}_{j=1}^J$*, investor* j*'s message function* $m_j(h_j):[0,+\infty]\to[0,+\infty]$*, for* $j=1,...,J$*, other agents' belief about investor* j*'s precision,* $\hat{h}_j(m_j):[0,\infty]\to[0,\infty]$*, for* $j=1,...,J$*, investors' trading strategies* $\{\alpha_j(m_1,...,m_J)\cdot\tilde{y}_j\}_{j=1}^J$*, and the market maker's pricing rule* $\lambda(m_1,...,m_J)\cdot\tilde{\omega}$*, such that:*

1. *On date 0:*

   (1.1) *Precision levels* $\left\{h_j^*\right\}_{j=1}^J$ *form a Nash equilibrium, as in Part (1) of Definition 1.*

   (1.2) *Given acquired precision* h_j*, investor* j*'s reported precision* $m_j(h_j)$ *maximizes her expected profits evaluated at the date-1 equilibrium, subject to* $m_j\le h_j$*. That is,*

$$m_j(h_j)=\arg\max_{m_j\le h_j}E\left[\alpha_j\tilde{y}_j\left(\tilde{v}-\lambda\alpha_j\tilde{y}_j-\lambda\sum_{k\ne j}^J\alpha_k\tilde{y}_k\right)-ch_j\right],\forall j,\quad\text{(S1)}$$

   where $\alpha_j\equiv\alpha_j(m_1(h_1^*),...,m_j,...m_J(h_J^*))$, $\lambda\equiv\lambda(m_1(h_1^*),...,m_j,...m_J(h_J^*))$, $\tilde{y}_j=\tilde{v}+\tilde{e}_j$ *with* $\tilde{e}_j\sim N\left(0,h_j^{-1}\right)$, $\tilde{y}_k=\tilde{v}+\tilde{e}_k$ *with* $\tilde{e}_k\sim N\left(0,h_k^{*-1}\right)$*, and* $\{\tilde{v},\tilde{e}_1,...,\tilde{e}_J\}$ *are mutually independent.*

2. *On date 1, investors' trading strategies and the market maker's pricing rule form a trading equilibrium that is characterized by Part (2) of Definition 1, and agents use the belief function* $\hat{h}_j(m_j)$ *to update their beliefs* $\hat{h}_j$ *about investor* j*'s information precision.*

3. *The belief functions satisfy the following properties:*

   (3.1) *Strictly increasing beliefs:* $\hat{h}_j(m_j)$ *is strictly increasing in* m_j, $\forall j$.

   (3.2) *Consistency:* $\hat{h}_j\left(m_j\left(h_j^*\right)\right)=h_j^*$, $\forall j$*; and if* $m_j(h_j)$ *is invertible at a particular* $h_j\neq h_j^*$*, then* $\hat{h}_j(m_j(h_j))=h_j$, $\forall j$.

Parts (1.1) and (2) of Definition S2 correspond respectively to Parts (1) and (2) of Definition 1, the equilibrium definition with exogenously given message functions. Part (1.2) of Definition S2 describes how investors optimally report their information-precision levels, which, in turn, endogenizes the message functions. Part (3.1) of Definition S2 is the strictly increasing belief criterion. Part (3.2) specifies consistency conditions that require beliefs to be consistent with the equilibrium strategies played by the agents. The first condition dictates that the beliefs must be consistent on the equilibrium path. The second condition encompasses consistency requirements about off-equilibrium paths: Whenever the precision m_j reported by investor j is implied by a unique actual precision level h_j according to the equilibrium message function $m_j(h_j)$, other agents can use and invert their knowledge of the message function to form belief $\hat{h}_j = h_j$.

Although the complete equilibrium definition above includes information-precision levels, message functions, belief functions, trading strategies, and the pricing rule, what really matters for allocations is information precision as well as the resulting trading and pricing rules. Thus, when two equilibria entertain the same information acquisition outcome and only differ in message functions or belief functions, we consider these two equilibria effectively the same equilibrium, as in the main text. In the subsequent analysis, we will focus on a symmetric equilibrium in which $h_j^* = h^*$ for $j = 1, ..., J$. Another advantage of imposing strictly increasing beliefs is that we obtain a unique symmetric equilibrium amount of information acquisition.

S2.2.1 Voluntary Disclosure by Monopolistic Investor

We begin analyzing voluntary disclosure by considering a setting with a monopolistic investor. As in Section 2.1, the investor's net expected trading profits $\pi(h, \hat{h})$ are given by equation (8). Taking the partial derivative of $\pi(h, \hat{h})$ with respect to the belief $\hat{h}$ yields the following, which reproduces inequality (18) in the main text:

$$\frac{\partial \pi(h, \hat{h})}{\partial \hat{h}} = -\frac{h\sigma_u}{4(1+h)\hat{h}^2}\sqrt{\frac{\hat{h}}{1+\hat{h}}} < 0.$$

This inequality implies that, given any actual precision level h, the monopolistic investor always wants the market maker's belief $\hat{h}$ about her signal precision to be the lowest possible. Hence, under any strictly increasing belief function $\hat{h}(m)$, the investor always reports $m = 0$ to minimize the market maker's belief $\hat{h}$, giving rise to the message function $m(h) = 0$.[3] In other words, the monopolistic investor conceals her information acquisition in equilibrium.

Closely related to the pricing effect discussed in Section 2, more disclosure about the investor's acquired information induces the market maker to believe she is faced with a more informed investor, and, thus, she sets a steeper pricing schedule to the detriment of the investor. As such, non-disclosure becomes the investor's optimal disclosure strategy, effectively making voluntary disclosure in a monopoly economy degenerate into the benchmark economy of secret information acquisition.

Lemma S1 (Alternative setup: Voluntary disclosure – Monopolistic investor)**.** *Consider a monopolistic investor who can choose to disclose her information-acquisition activity. There exists a unique equilibrium in which the investor engages in secret information acquisition and acquires private information of precision h_s, where h_s is the unique solution to equation (9) in Part (1) of Lemma 1. The investor's trading strategy and the market maker's pricing rule are accordingly given by Part (1) of Lemma 1.*

S2.2.2 Voluntary Disclosure by Oligopolistic Investors

This section studies a setting with oligopolistic investors ($J \geq 2$). As in Section 2.2, we introduce the interaction among strategic investors and investigate its interplay with the pricing effect. The following lemma extends Lemma S1 to the general case where $J \geq 2$ and characterizes the unique symmetric equilibrium under extreme conditions. The numerical analysis in Figure S3 demonstrates its generality.

Lemma S2 (Alternative setup: Voluntary disclosure – Oligopolistic investors)**.** *Consider oligopolistic investors who can disclose their information-acquisition activities.*

[3] In Lemma S1, we do not explicitly specify a belief function $\hat{h}(m)$ since its specific form is not important in determining the equilibrium information precision h_s. In fact, an infinite number of belief functions support the equilibrium information precision h_s. For instance, $\hat{h}(m) = h_s + \beta m$, where $\beta > 0$.

(1) *When c/σ_u is sufficiently high, there exists a unique symmetric equilibrium in which all investors engage in secret information acquisition and acquire private information of precision h_s, where h_s is the unique solution to equation (12). The investor's trading strategy and the market maker's pricing rule are accordingly given by Part (1) of Lemma 2.*

(2) *When c/σ_u is sufficiently low, there exists a unique symmetric equilibrium in which all investors engage in overt information acquisition and acquire private information of precision h_o, where h_o is the unique solution to equation (13). The investor's trading strategy and the market maker's pricing rule are accordingly given by Part (2) of Lemma 2.*

Lemma S2 states that the equilibrium for voluntary disclosure in the oligopolistic setting crucially hinges on the ratio c/σ_u of information-acquisition cost c to the volatility σ_u of noise trading. Specifically, when c/σ_u is sufficiently high, investors do not reveal their information-acquisition activities and the equilibrium in the voluntary disclosure game is the same as that of secret information acquisition. By contrast, when c/σ_u is sufficiently low, all investors fully reveal their information-acquisition activities and the equilibrium becomes the same as that of overt information acquisition.

Using equations (14) through (16) in Section 2.2, we can express investor j's expected net trading profits π_j as functions of $(h_j, \hat{h}_j; h^*)$, where h_j is investor j's information precision, $\hat{h}_j$ is other agents' belief about h_j, and h^* is the equilibrium information precision chosen by other $J-1$ investors. When c/σ_u is sufficiently high, the partial derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ with respect to $\hat{h}_j$ can be shown to be negative, regardless of the level of investor j's actual precision h_j. That is,

$$\lim_{c/\sigma_u \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} < 0,$$

which is the same as inequality (19) in the main text. As a result, similar to the monopolistic-investor setting, investor j reports $m_j = 0$ under any strictly increasing belief function $\hat{h}_j(m_j)$. Intuitively, with high c/σ_u, the competition among investors is mild since none of them produces precise information, and the interaction between investors and the market maker is the driving force in determining investors' disclosure incentives. In a sense, each investor is like a local monopolist of her own information since each's imprecise private information is very different from the other's; hence, the equilibrium in the oligopolistic setting with high c/σ_u also resembles that in the monopolistic setting.

By contrast, when c/σ_u is sufficiently low, investors fully reveal their information precision. Regardless of h_j, the partial derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ with respect to $\hat{h}_j$ becomes positive,

$$\lim_{c/\sigma_u \to 0} \frac{\partial \pi_j(h_j, \hat{h}_j; h^*)}{\partial \hat{h}_j} > 0,$$

which is the same as inequality (20) in the main text. Hence, under a strictly increasing belief function $\hat{h}_j(m_j)$, investor j chooses to report $m_j = h_j$. This implies an endogenous message function $m_j(h_j) = h_j$ and an endogenous belief function $\hat{h}_j(m_j) = h_j$, which effectively makes the investor an overt investor. Intuitively, with low c/σ_u, investors can acquire very accurate information so that competition among them intensifies. In this case, given her information precision, each investor is keen to build up a competitive advantage by fully disclosing her information precision and forcing her rivals to scale back in their informed trading. As such, the equilibrium degenerates to that in the benchmark economy with overt information acquisition.

Overall, the pricing effect prevails when c/σ_u is high, leading to secret information acquisition whereas the competition effect dominates when c/σ_u is low and results in overt information acquisition.

We further confirm the intuition gleaned from Lemma S2 using numerical analyses. Figure S3 plots information precision against c/σ_u in a duopoly setting ($J = 2$). Since the equilibrium features either secret or overt information acquisition, we plot information precision h_o and h_s using blue and red curves, respectively. The equilibrium value h^* of information precision is displayed in the solid curve. We also use shaded areas to indicate whether $h^* = h_o$ or $h^* = h_s$. Consistent with Lemma S2, Figure S3 shows that, in equilibrium, when c/σ_u is lower than a threshold, investors engage in overt information acquisition whereas, when c/σ_u is higher than that threshold, investors engage in secret information acquisition.

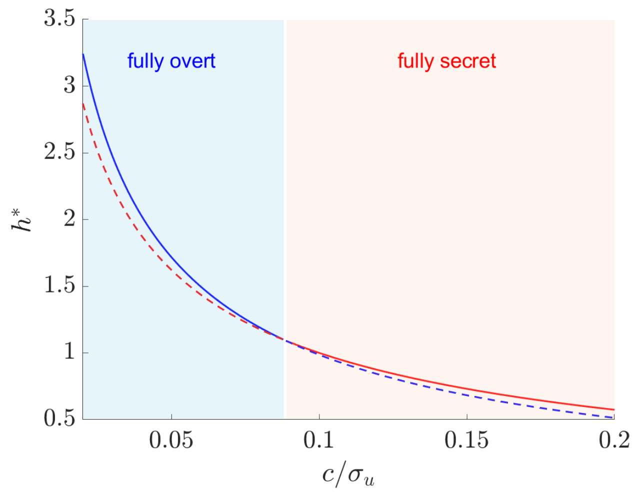

Figure S3: Alternative setup: Information acquisition under voluntary disclosure in a duopoly

S2.3 Model Implications

Incidence of voluntary disclosure of site visits Using Lemmas S1 and S2 and Figure S3, we obtain the following results regarding the incidence of voluntary disclosure in this alternative model of site visits, which are similar to the results of the main text's Proposition 3. First, the common prior that funds are secretive about their research activities applies to our model when J is small or when c/σ_u is high (see Lemma S1 and Part (1) of Lemma S2). When the cost of conducting site visits is very high (i.e., high c) or when the firm is mainly traded by institutional investors (i.e., low σ_u), funds indeed go to great lengths to hide their research activities and deter the market maker's steep pricing schedule. However, when J is large and c/σ_u is low, funds adopt an opposite disclosure strategy and fully disclose their research activities (see Part (2) of Lemma S2). Disclosing funds gain advantages by making the competing investors trade less aggressively, which could potentially explain why some funds voluntarily disclose their corporate site visits.

Proposition S1 (Alternative setup: Incidence of voluntary disclosure)**.**

(1) *In the monopolistic or oligopolistic investor setting with sufficiently high c/σ_u, investors never voluntarily disclose their site visits in the presence of arbitrarily small*

disclosure costs.

(2) *In the oligopolistic investor setting with sufficiently low c/σ_u, investors voluntarily disclose their site visits.*

Effectiveness and consequences of mandatory disclosure of site visits Using Lemmas S1 and S2, we obtain the following proposition, which is similar to Proposition 4 in the main text.

Proposition S2 (Alternative setup: Effectiveness of mandatory disclosure)**.**

(1) *In the monopolistic-investor setting, mandatory disclosure of site visits is effective.*

(2) *In the oligopolistic-investor setting, mandatory disclosure of site visits is effective when c/σ_u is sufficiently high.*

(3) *When effective, mandatory disclosure causes investors to acquire less precise information and improves their payoff and market liquidity but worsens market efficiency.*

Based on Lemma S1, the disclosure mandate is always effective under the monopoly setting, which is Part (1) of Proposition S2. Under the oligopolistic setting, as shown in Lemma S2 and Figure S3, investors voluntarily disclose their information-acquisition activities when c/σ_u is low. Therefore, regulations that mandate the disclosure of information-acquisition activities are redundant, which highlights Part (2) of Proposition S2. Part (3) of Proposition S2 also suggests that, when c/σ_u is sufficiently high, mandatory disclosure can weakly Pareto-improve the well-being of all financial market participants, as discussed in the main text.

Discussion This appendix's modeling approach, which is also seen in Shin (2003), is simpler and seems a more natural extension of standard disclosure models. However, we do not take this route in our main text because, in reality, institutional investors rarely conduct multiple visits to the same firm within the same year. For instance, Cheng, Du, Wang, and Wang (2019, p.366) show that the average frequency of site visits per firm ranges from 5.78 in 2013 to 9.77 in 2010. They further investigate the visitors' identities and find that, in general, the same visitor does not visit the same company more than once a year; in fact, only 12.5 percent of site visits are conducted by those who have visited the same firm in the same year. To reflect reality as closely as possible, our main text has adopted the approach with an exogenous $\bar{h}$ to model the site-visits context.

S2.4 Proofs

Proof of Lemma S1

Following the same derivation as in Section 2.1.1, the monopolistic investor's expected net trading profit is given by (8). Taking the derivative of $\pi(h, \hat{h})$ with respect to $\hat{h}$ yields $\frac{\partial \Pi(h,\hat{h})}{\partial \hat{h}} = -\frac{h\sigma_u}{4(1+h)\hat{h}^2}\sqrt{\frac{\hat{h}}{1+\hat{h}}} < 0$. Under strictly increasing beliefs, this inequality suggests that regardless of the monopolistic investor's actual information acquisition, the investor tends to disclose the lowest possible level of signal precision, giving rise to $m = 0$.

Next we solve for the investor's optimal information-acquisition strategy. Since the monopolistic investor withholds her information acquisition, the market maker only holds the belief that the investor acquires the equilibrium level h_s of information. Setting $\hat{h} = h_s$ in (8), the monopolistic investor's expected net trading profit becomes $\pi(h, h_s) = \frac{h\sigma_u\sqrt{1+h_s}}{2(1+h)\sqrt{h_s}} - ch$. Maximizing the profit with respect to h and then setting $h = h_s$ yields the optimal precision h_s, which is the unique solution to the equation (9). We also confirm the second-order condition (SOC): $\frac{\partial^2\pi}{\partial h^2} = -\frac{\sigma_u}{(1+h)^3\sqrt{\frac{h_s}{1+h_s}}} < 0$.

Overall, an equilibrium with strictly increasing beliefs must be the one in which the monopolistic investor engages in secret information acquisition and the equilibrium information precision is h_s. One example of the beliefs that sustain the equilibrium is $\hat{h}(m) = h_s + m$. Under this belief, we obtain the investor's expected net trading profits (8) as follows:

$$\pi(h, m) = \frac{h\sigma_u\sqrt{1 + h_s + m}}{2(1 + h)\sqrt{h_s + m}} - c \cdot h.$$

Given h, maximizing the net profit yields the optimal message $m = 0$. Then, following the same procedure as above, we confirm that given the disclosure strategy, the investor will not deviate from h_s in information acquisition.

Proof of Lemma S2

Following the same derivation as in the proof of Lemma 2, investor j's expected net trading profit is given by (A5). For simplicity, we focus on the parameter c and discuss the two cases $c \to \infty$ and $c \to 0$, which are effectively $c/\sigma_u \to \infty$ and $c/\sigma_u \to 0$.

Case 1. Sufficiently high c. We conjecture that when c is sufficiently high, in equilibrium investors engage in secret information acquisition; that is, $h^* = h_s$. Setting $h^* = h_s$ in $\pi_j(h_j, \hat{h}_j; h^*)$ as given by (A5), taking the derivative of $\pi_j(h_j, \hat{h}_j; h^*)$ with respect to the belief $\hat{h}_j$, and imposing $h_s \to 0$ we obtain that

$$\lim_{h_s \to 0} \frac{\partial \pi_j(h_j, \hat{h}_j; h_s)}{\partial \hat{h}_j} = -\frac{h_j(1+\hat{h}_j)\sigma_u}{4(1+h_j)(\hat{h}_j(1+\hat{h}_j))^{3/2}} < 0. \tag{SA1}$$

Furthermore, based on (12), we know that $\lim_{c\to\infty} h_s = 0$. Coupled with (SA1), we thus show that

$$\lim_{c \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h_s)}{\partial \hat{h}_j} < 0. \tag{SA2}$$

That is, when c is sufficiently high, the lower the other agents' belief about investor j's signal precision, the higher investor j's profit. Therefore, regardless of her actual information acquisition, when c is sufficiently high, investor j always wants the other agents' beliefs about her signal precision to be the lowest possible. Since we focus on strictly increasing beliefs, to achieve the lowest possible belief, investor j can set $m_j = 0$.

Next, having shown that investor j's $m_j = 0$ is sustained regardless of h_j, we will prove that given this message function, investor j will not deviate from the equilibrium information-acquisition strategy h_s. Since investor j does not reveal her signal precision, even if she deviates from the equilibrium information-acquisition strategy, other agents (including the other investors and the market maker) still believe that her signal precision is maintained at h_s, i.e., $\hat{h}_j = h_s$. Replacing $\hat{h}_j = h_s$ in (A5) we can compute investor j's net trading profit as given by (A6). Taking the derivative of the net trading profit with respect to h_j and imposing $h_j = h_s$ yields the following:

$$\frac{\partial \pi(h_j, h_s; h_s)}{\partial h_j}|_{h_j=h_s} = \frac{\sigma_u}{(2 + h_s + h_s J)\sqrt{h_s(1 + h_s)J}} - c = 0,$$

where the second equality follows equation (12). We also confirm that the SOC of $\pi(h_j, h_j; h_s)$ with respect to h_j is negative:

$$\frac{\partial^2 \pi(h_j, h_s; h_s)}{\partial h_j^2} = -\frac{2(1 + h_s)^2 \sigma_u}{(1 + h_j)^3(2 + h_s + h_s J)\sqrt{h_s(1 + h_s)J}} < 0.$$

Therefore, $h_j = h_s$, where h_s is the unique solution to equation (12), indeed maximizes investor j's expected profit net of the information-acquisition cost. In other words, investor j will not deviate from the equilibrium information-acquisition strategy.

Case 2. Sufficiently low c. We conjecture that when c is sufficiently low, in equilibrium investors engage in overt information acquisition; that is, $h^* = h_o$. Setting $h^* = h_o$ in (A5), taking the derivative with respect to the belief $\hat{h}_j$, and imposing $h_o \to \infty$ yields the following:

$$\lim_{h_o \to \infty} \frac{\partial \pi_j(h_j, \hat{h}_j; h_o)}{\partial \hat{h}_j} = \frac{h_j(1 + \hat{h}_j)\sigma_u \cdot \Psi_1(\hat{h}_j)}{2(1 + h_j)(\hat{h}_j + \hat{h}_j J + 2J)^2 \left((J - 1)(3\hat{h}_j + 4) + \hat{h}_j(1 + \hat{h}_j J)\right)^{3/2}} > 0.$$

The positive sign holds because $\Psi_1(\hat{h}_j) = \hat{h}_j^2(4J^2 - 3J - 3) + \hat{h}_j(16J^2 - 21J + 1) + (16J^2 - 26J + 8) > 0$, where the inequality holds because the discriminant $(16J^2 - 21J + 1)^2 - 4(4J^2 - 3J - 3)(16J^2 - 26J + 8) < 0$ and when $J \geq 2$, $4J^2 - 3J - 3 > 0$. That is, $\Psi_1(\hat{h}_j)$ opens upward and does not interact with the x-axis.

Next, equation (13) implies that if its right-hand side (RHS) diverges to ∞ due to sufficiently low c, h_o must diverge to ∞ as well, i.e., $h_o = O(c^{-1/4})$, where the notation $X_2 = O(X_1)$ means that $\frac{X_2}{X_1}$ converges to a finite constant as $c \to 0$. Coupled with the result $\lim_{h_o\to\infty} \frac{\partial \pi_j(h_j,\hat{h}_j;h_o)}{\partial \hat{h}_j} > 0$, we know that

$$\lim_{c\to 0} \frac{\partial \Pi_j(h_o, h_j, \hat{h}_j)}{\partial \hat{h}_j} > 0.$$

That is, investor j's expected profit is monotonically increasing in the belief $\hat{h}_j$. Therefore, regardless of the actual information acquisition h_j, investor j always wants other agents' belief $\hat{h}_j$ about her information acquisition the highest possible. So, when choosing disclosure strategy, investor j would like the message that induces the highest possible belief, which results in $m_j = h_j$ under strictly increasing beliefs.

We next confirm that investor j will not deviate from the equilibrium information acquisition h_o. Replacing $\hat{h}_j = h_j$ and $h^* = h_o$ in equation (A5) yields investor j's expected net trading profit $\pi_j(h_j, h_j; h_o)$ as given by (A7). Taking the derivative of $\pi_j(h_j, h_j; h_o)$ with respect to h_j and imposing $h_j = h_o$ yields

$$\frac{\partial \pi_j(h_j, h_o)}{\partial h_j}|_{h_j=h_o} = \frac{\left((6J^2 - J - 3)h_o^2 + (4J^2 + 10J - 8)h_o + 8J - 4\right)\sigma_u}{2J(2 + h_o)(2 + h_o + h_o J)^2\sqrt{h_o(1 + h_o)}} - c = 0,$$

where the last equality follows from equation (13). The SOC of $\pi_j(h_j, h_j; h_o)$ with respect to h_j at $h_j = h_o$ is

$$\frac{\partial^2 \pi_j(h_j, h_j; h_o)}{\partial h_j^2}|_{h_j=h_o} = -\frac{(2 + h_o)\sigma_u \cdot \Psi_2(h_o)}{4J(2 + h_o + h_o J)^3 \left(h_o(1 + h_o)(2 + h_o)^2 J\right)^{3/2}} < 0.$$

The negative sign follows because

$$\begin{aligned}\Psi_2(h_o) = {} & 64J - 48 + 16(8J^2 + 11J - 12)h_o \\ & + 4h_o^2(12J^3 + 105J^2 + 10J - 66) + 4h_o^3(52J^3 + 85J^2 - 43J - 36) \\ & + h_o^4(24J^4 + 184J^3 + 21J^2 - 102J - 27) + 4h_o^5 J(6J^3 + 5J^2 - 4J - 3) > 0,\end{aligned}$$

where the inequality holds because the coefficients of the constant term $64J - 48$ and those of all h_o terms are positive when $J \geq 2$. Therefore, $h_j = h_o$ indeed maximizes

$\pi_j(h_j, h_j; h_o)$, that is, investor j will not deviate from the equilibrium information acquisition h_o.

Proof of Proposition S1

This proposition immediately follows Lemmas S1 and S2.

Proof of Proposition S2

Most proof of the proposition immediately follows Lemmas S1 through S2 and Propositions 1 through 2. We here only prove that when c/σ_u is sufficiently high, investors' expected net trading profits increase upon the disclosure mandate. Given an equilibrium level h of information acquisition, we can derive investors' expected net trading profit as follows by replacing $h^* = h_j = \hat{h}_j = h$ in equation (A5): $\pi(h) = \frac{\sqrt{h(1+h)}\sigma_u}{\sqrt{J}(2+h+hJ)} - c \cdot h$. Taking the derivative of $\pi(h)$ with respect to h yields

$$\frac{\partial \pi(h)}{\partial h} = \sigma_u \left(\frac{2 - h(J-3)}{2(2+h+hJ)^2\sqrt{h(1+h)J}} - \frac{c}{\sigma_u} \right). \tag{SA3}$$

For sufficiently high c/σ_u, based on equations (12) and (13) we know that both $h_o \to 0$ and $h_s \to 0$. Setting $h \to 0$ in (SA3) yields $\frac{\partial \pi(h)}{\partial h} = \sigma_u \left(\frac{1}{4J\sqrt{h}} - \frac{c}{\sigma_u} \right)$. Thus, we know that $\frac{\partial \pi}{\partial h} < 0$ if and only if

$$\sqrt{h} > \frac{1}{4J}\frac{\sigma_u}{c}. \tag{SA4}$$

Based on equations (A9) and (A10) in the proof of Proposition 2, we know that when c/σ_u is sufficiently high, $\sqrt{h_o} = \frac{1}{2\sqrt{J}}(1 - \frac{1}{2J}) \cdot \frac{\sigma_u}{c} + o\left(\frac{1}{c}\right) > \frac{1}{4J}\frac{\sigma_u}{c}$ and $\sqrt{h_s} = \frac{1}{\sqrt{J}} \cdot \frac{\sigma_u}{c} + o\left(\frac{1}{c}\right) > \frac{1}{4J}\frac{\sigma_u}{c}$, where the two inequalities follow because $J > 2$. Furthermore, we've shown in Proposition 2 that for sufficiently high c/σ_u, $h_s > h_o$; coupled with $\frac{\partial \pi}{\partial h} < 0$ we know that $\pi_s < \pi_o$; that is, from the secret to overt case, investors' expected net trading profit improves.

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